MONDAY 3RD NOVEMBER : MAXIMIZING AND MINIMIZING

Reading: sections 11.7 and 11.8
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Tangent planes and normal lines to surfaces

(1) Find an equation for the tangent plane to the surface

$$xe^{yz} = 1$$

at the point $(1, 0, 5)$.

(2) Find points on the hyperboloid

$$x^2 - y^2 + 2z^2 = 1$$

where the normal line is parallel to the line through the points $(3, -1, 0)$ and $(5, 3, 6)$.

2. Maximizing and minimizing

(1) Find and classify the critical points of

$$f(x, y) = x^3y + 12x^2 - 8y$$

(2) Find and classify the critical points of

$$f(x, y) = xye^{-x^2-y^2}$$

3. Absolute maximum and minimum values

(1) Find the maximum value of

$$f(x, y) = 3x - 2y + 5$$

on the (closed) triangle with vertices $(0, 0)$, $(3, 1)$ and $(2, 2)$.

(2) Find the maximum value of

$$g(x, y) = x^2 + y^2 + x + y$$

on the disc $x^2 + y^2 \leq 1$.

4. A harder problem

(1) (a) Solve the first problem in section 3 by sketching level curves of f
(b) Solve the second problem in section 3 by sketching level curves of g
(c) Write down equations that need to be satisfied if the level curve of g and the level curve of h are tangent to each other at the point (a, b). Use this to find the maximum and minimum values of g on the circle $x^2 + y^2 = 1$. (g is defined in the second problem in section 3.)