WEDNESDAY 5TH NOVEMBER : MAXIMIZING AND MINIMIZING II

Reading: sections 11.7 and 11.8
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Maximizing and minimizing on regions with boundary

(1) Suppose that we want to find the maximum and minimum values of

$$f(x, y) = x^2 + y^2 + xy^2$$

on the square with vertices $(3, 3)$, $(3, -3)$, $(-3, 3)$, and $(-3, -3)$. We can approach this as follows.

(a) First find interior maxima and minima. These occur at critical points, so find the critical points of f. Are they local maxima, local minima or saddle points?

(b) Now find the maxima and minima on the boundary. The boundary is made up of four curves. Parametrize each curve in turn, and hence find the maximum and minimum values of f along that curve.

(c) What are the global maximum and minimum values of f on the square?

2. Maximizing and minimizing subject to constraints

(1) Find the maximum and minimum values of

$$f(x, y) = x^2 + y^2$$

subject to the constraint

$$x^4 + y^4 = 1$$

(2) Find the points on the surface $x^2y^2z = 1$ which are closest to the origin.

(3) Find the dimensions of the rectangular box of maximum volume which has surface area 64 cm^2.