

FRIDAY 7TH NOVEMBER : LAGRANGE MULTIPLIERS

Reading: sections 12.1 and 12.2
Homework: see www.courses.fas.harvard.edu/~math21a/

1. CONSTRAINED OPTIMIZATION PROBLEMS

- (1) Find the volume of the largest rectangular box in the first octant with three faces in the co-ordinate planes and one vertex on the plane

$$2x + 3y + 5z = 7$$

- (2) A rectangular box without a lid must have a volume of 1 cubic meter. What dimensions minimize the amount of cardboard used?

- (3) (a) Extend the argument from last time to show that if we want to maximize or minimize a function $f(x, y, z)$ subject to the constraints

$$g(x, y, z) = k_1 \quad \text{and} \quad h(x, y, z) = k_2$$

then we should solve

$$\nabla f = \lambda \nabla g + \mu \nabla h$$

$$g(x, y, z) = k_1$$

$$h(x, y, z) = k_2$$

- (b) Maximize

$$f(x, y, z) = x + 2y$$

subject to the constraints

$$x + y + z = 1 \quad y^2 + z^2 = 4$$

2. REVIEW PROBLEMS

- (1) Find the points on the sphere $x^2 + y^2 + z^2 = 9$ where the tangent plane is parallel to the plane $4x + y = 5$.

- (2) Find the maximum rate of change of the function

$$f(x, y) = x^3y + \sqrt{xy}$$

at the point $(2, 1)$. In what direction does it occur? What is the rate of change of f in the direction $(1, 1)$?

- (3) Suppose that the radius of a cylinder is increasing at the rate of 1 cm/s and the height is decreasing at the rate of 2 cm/s. How fast is the volume changing when the radius and the height are both 2m?

- (4) Find the absolute maximum and minimum values of

$$f(x, y) = e^{-x^2-y^2}(3x^2 - y^2)$$

on the disk $x^2 + y^2 \leq 16$.