

**FRIDAY 10TH OCTOBER : DIFFERENTIATING VECTOR-VALUED
FUNCTIONS**

Reading: sections 10.2 and 10.3
Homework: see www.courses.fas.harvard.edu/~math21a/
note that the first computing assignment is due next Wednesday

1. COMPUTING TANGENT LINES

- (1) Find the tangent lines to the curves with given parametrizations at the points indicated:
- (a) $\mathbf{r}(t) = e^t \mathbf{i} + e^{-2t} \mathbf{j}$, $(1, 1)$
 - (b) $\mathbf{r}(t) = \langle \ln t, 2\sqrt{t}, t^2 \rangle$, $(0, 2, 1)$

- (2) Consider the point $P(t)$ with position vector

$$\langle \cos t, \sin t, t \rangle$$

As t varies, this traces out a curve in \mathbb{R}^3 called a helix. Find the tangent line L to the helix at the point $P(t)$. Let $Q(t)$ be the point at which L meets the xy -plane. As t varies, the point $Q(t)$ traces out a curve. Sketch this curve. Then draw a 3D picture of the helix, and draw on a couple of tangent lines and the curve.

2. DIFFERENTIATING VECTOR-VALUED FUNCTIONS

- (1) Suppose that a particle is moving on the surface of a sphere of radius 4 about the origin. Show that the velocity of the particle is always perpendicular to its position. (Hint: let $\mathbf{r}(t)$ be the position of the particle at time t , and differentiate $\mathbf{r}(t) \cdot \mathbf{r}(t)$)
- (2) Conversely, show that if the velocity of a particle is always perpendicular to the velocity, then the particle moves on a sphere.

3. A MORE CHALLENGING PROBLEM : ANGULAR MOMENTUM

- (1) Write down Newton's Second Law of Motion¹. (Your answer should involve vectors!)

- (2) Suppose that a particle is moving under the influence of a radial force. This means that if the particle is at position $\mathbf{r}(t)$ at time t , then the force on it at time t is parallel to $\mathbf{r}(t)$. (For example, the particle could be moving under the gravitational attraction of a planet centered at the origin.) Show that the *angular momentum*

$$\mathbf{h} = \mathbf{r}(t) \times \mathbf{r}'(t)$$

is conserved. In other words, show that

$$\frac{d}{dt}(\mathbf{h}) = 0$$

- (3) Deduce that the particle moves in a plane.

¹If you don't know what this is, ask the person sitting next to you. If that fails, look at the top of page 728 of your textbook.