WEDNESDAY 15TH OCTOBER : ARC LENGTH / PARAMETRIZING SURFACES

1. ARC LENGTH

(1) Sketch the segment of helix given by
$$\mathbf{r}(t) = \langle \sin t, \cos t, 3t \rangle \qquad 0 \le t \le \pi$$

Find its length.

(2) Let $s(T)$ be the length of the segment of curve
$$\mathbf{r}(t) = e^t \cos t \mathbf{i} + e^t \sin t \mathbf{j}$$

where $-\infty \le t \le T$.
  (a) Compute $s(T)$
  (b) Express t in terms of $s = s(t)$
  (c) Give an *arc-length parametrization* of the curve — in other words, give a parametrization of the curve in terms of the variable s.

2. Parametrizing surfaces

Give parametrizations of the following surfaces:

(1) The parabolic cylinder with equation $y = x^2$

(2) The part of the parabolic cylinder with equation $y = x^2$ which lies in front of the plane $x = 0$

(3) The sphere of radius 1 about the origin