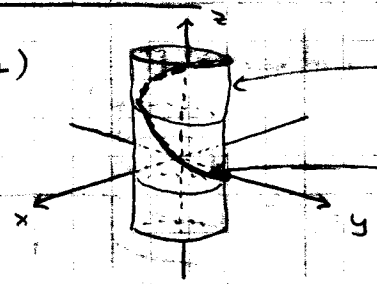
SOLUTIONS : ARC LENGTH, PARAMETRIZING SURFACES

SECTION ONE

(1)



the cylinder $x^2+y^2=1$, on which the helix lies

the helix

$$\underset{\sim}{r}'(t) = \langle \cos t, -\sin t, 3 \rangle$$

$$\text{Length of helix} = \int_{t=0}^{t=\pi} \|\underset{\sim}{r}'(t)\| \, dt$$

$$= \int_0^{\pi} \sqrt{\cos^2 t + \sin^2 t + 9} = \underline{\underline{\pi\sqrt{10}}}$$

(2) $\underset{\sim}{r}'(t) = \langle e^t \cos t - e^t \sin t, \; e^t \sin t + e^t \cos t \rangle$

$$\text{so} \quad s(t) = \int_{-\infty}^{t} \|\underset{\sim}{r}'(t)\| \, dt$$

$$= \int_{-\infty}^{t} \sqrt{2e^{2t}\cos^2 t + 2e^{2t}\sin^2 t} \, dt$$

$$= \sqrt{2} \int_{-\infty}^{t} e^t \, dt = \underline{\underline{\sqrt{2}\, e^t}}$$

Thus $t = \ln(s/\sqrt{2})$

and the arc-length reparametrization is

$$\underline{\underline{\underset{\sim}{r}(s) = \left\langle \frac{s}{\sqrt{2}} \cos\left(\ln\left(\frac{s}{\sqrt{2}}\right)\right), \; \frac{s}{\sqrt{2}} \sin\left(\ln\left(\frac{s}{\sqrt{2}}\right)\right) \right\rangle}}$$

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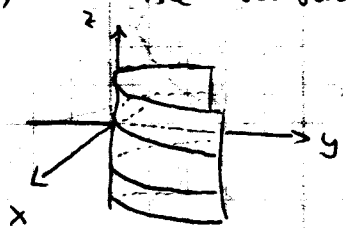
SECTION TWO

(1) The surface $y = x^2$ looks like



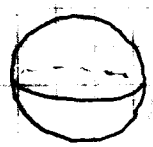
One parametrization is given by

$$x = u$$
$$y = u^2$$
$$z = v \qquad \text{for} \quad -\infty \le u, v \le \infty$$

i.e. $\underset{\sim}{r}(u,v) = \langle u, u^2, v \rangle$ and the domain of the parametrization is (u,v) in all of $\mathbb{R}^2$.

(2) The part of the surface $y = x^2$ in front of the plane $x = 0$ can be parametrized by the same function $\underset{\sim}{r}(u,v)$ but the domain restricted to $\{u \geq 0, \ v \in \mathbb{R}\}$.

(3) To parametrize the sphere of radius 1 about the origin, we can use spherical polar co-ordinates as a guide:



In spherical polars this is $\rho = 1$
$0 \le \theta < 2\pi$
$0 \le \phi \le \pi$

so take $\rho = 1$
$\theta = u$
$\phi = v$ for $0 \le u < 2\pi$
$0 \le v \le \pi$

or in other words

$$\underset{\sim}{r}(u,v) = \langle \sin v \cos u, \sin v \sin u, \cos v \rangle \qquad 0 \le u \le 2\pi, \ 0 \le v < \pi$$