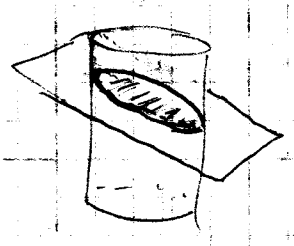
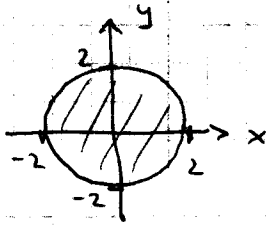
SOLUTIONS: PARAMETRIZING SURFACES (CONTINUED)

SECTION ONE

(1)



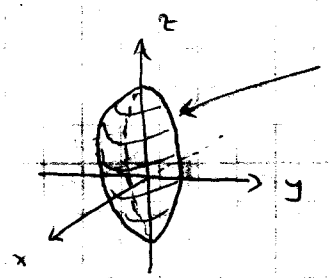
Projecting to the xy-plane we see the disc $x^2+y^2 \leq 4$



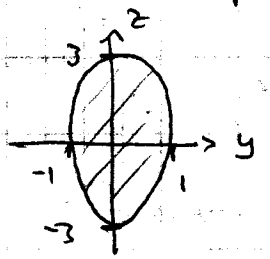
Let the parametrization be $x = u,\ y = v,\ z = 4-3u-2v$

where (u,v) lies in the disc $D = \{(u,v) : u^2+v^2 \leq 4\}$

(2)



Projecting to the yz-plane we see an ellipse (filled in):



The region (in the yz plane) is $y^2 + \frac{z^2}{9} \leq 1$

So take the parametrization to be $x = 2\sqrt{1-y^2-z^2/9}$

$y = u$

$z = v$

where (u,v) lies in the region $D = \{(u,v) : u^2 + \frac{v^2}{9} \leq 1\}$

(3) Rotating $x = \sqrt{z}$ about the z-axis gives $r = \sqrt{z}$ (in cylindrical polars) i.e. $z = x^2+y^2$.

The parametrization of the paraboloid $z = x^2+y^2$ which this suggests is

$x = \sqrt{z}\cos\theta$

$y = \sqrt{z}\sin\theta$

$z = z$

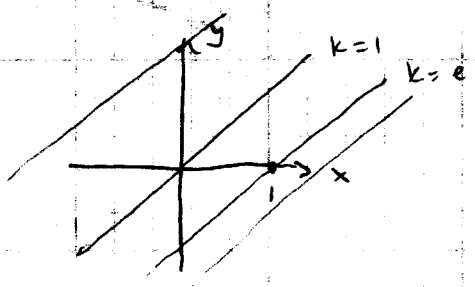
for $\begin{cases} 0 \leq \theta < 2\pi \\ 0 \leq z \end{cases}$

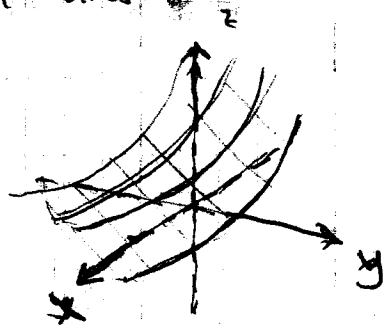
SOLUTIONS : FUNCTIONS OF SEVERAL VARIABLES

SECTION TWO

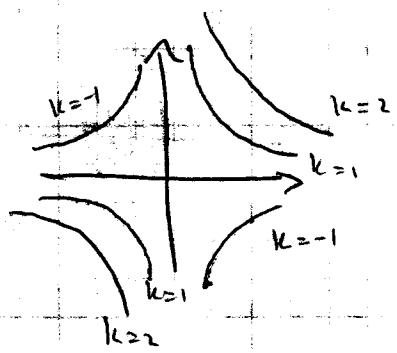
1 (a) $e^{x-y} = k \iff x - y = \ln k \iff y = x - \ln k$

so the level curves are straight lines :

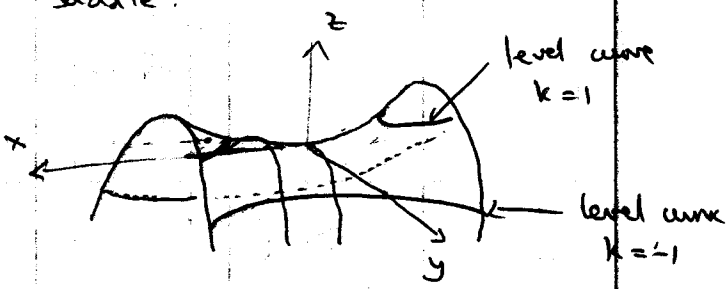




(b) $xy = k$ is a hyperbola : $y = k/x$



The graph looks like a saddle :



2 (a) These are planes perpendicular to $\langle 1, 2, 3 \rangle$

(b) These are spheres with center the origin