SOLUTIONS: LINES AND PLANES

SECTION 1

(1) $\underset{\sim}{r} = \langle 2,0,-1\rangle + t\langle -1,2,2\rangle$

(2) The direction vectors of the lines are $\langle 1,-2,4\rangle$ and $\langle -1,4,-3\rangle$. These are not parallel, so the lines are not parallel.

Solving $\begin{cases} t-2 = 1-s \\ 4-2t = 4s-4 \\ 4t-8 = 3-3s \end{cases}$

gives $t=2$, $s=1$ so they meet.

To find where they meet, substitute $t=2$ to get $\underline{\underline{\langle 0,0,0\rangle}}$.

(3) The direction vectors (which are $\langle 1,-1,4\rangle$ and $\langle 1,1,3\rangle$) are not parallel, so the lines are not parallel.

Solving $\begin{cases} t-2 = s+1 \\ 4-t = s-1 \\ 4t-8 = 3s+2 \end{cases}$

gives no solutions, so the lines do not meet.

They are skew.

Solutions: Lines and Planes

Section 2

(1) Vector perpendicular to the plane is $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$\overrightarrow{PQ} = \langle -1, +1, 0 \rangle$

$\overrightarrow{PR} = \langle -1, 0, 2 \rangle$

so $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 1 & 0 \\ -1 & 0 & 2 \end{vmatrix} = \langle 2, 2, 1 \rangle$

An equation for the plane is therefore

$$\underline{\underline{(\langle x,y,z \rangle - \langle 1,1,0 \rangle) \cdot \langle 2,2,1 \rangle = 0}}$$

or in other words $\underline{\underline{2x + 2y + z = 4}}$

(2) Plugging $x = 2-t$, $y = 2t$, $z = 2t-1$

into $2x + 2y + z = 4$ we find $t = 1/4$

so they meet at $\underline{\underline{(7/4, 1/2, -1/2)}}$