

MONDAY 20TH OCTOBER : LIMITS AND CONTINUITY

Reading: sections 11.2 and 11.3

Homework: Note that the homework which was due on Wednesday is now due on Monday
— see www.courses.fas.harvard.edu/~math21a/ for the revised assignment schedule.

1. NON-EXISTENCE OF LIMITS

(1) Consider the function

$$f(x, y) = \frac{xy}{x^2 + y^2}$$

- (a) Compute the limit of $f(x, y)$ as (x, y) approaches $(0, 0)$ along the x -axis.
- (b) Compute the limit of $f(x, y)$ as (x, y) approaches $(0, 0)$ along the line $y = x$.
- (c) Does

$$\lim_{(x,y) \rightarrow (0,0)} f(x, y)$$

exist?

(2) Let

$$f(x, y) = \frac{3x^2y}{x^4 + y^2}$$

- (a) Compute the limit of $f(x, y)$ as (x, y) approaches $(0, 0)$ along the line $y = mx$.
- (b) Compute the limit of $f(x, y)$ as (x, y) approaches $(0, 0)$ along the line $y = x^2$.
- (c) Does

$$\lim_{(x,y) \rightarrow (0,0)} f(x, y)$$

exist?

2. A USEFUL TOOL: POLAR CO-ORDINATES

(1) Use polar co-ordinates either to compute the following limits, or to show that they do not exist.

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y}{\sqrt{x^2 + y^2}}$$

(b)

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{\sin(x^2 + y^2 + z^2)}{x^2 + y^2 + z^2}$$

(c)

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^2 - y^2 + z^2}{x^2 + y^2 + z^2}$$

3. SOME HARDER QUESTIONS

(1) For each of the following functions, either prove that the function is continuous everywhere or find a point of discontinuity and prove that it is discontinuous there.

(a)

$$f(x, y) = \frac{x^3y^2}{x^4 + y^8}$$

(b)

$$f(x, y, z) = \frac{x + y + z}{e^{x^2+y^2+z^2}}$$