WEDNESDAY 22ND OCTOBER : PARTIAL DERIVATIVES

Reading: sections 11.3 and 11.4
Homework: Note that the homework which was due today is now due on Monday — see `www.courses.fas.harvard.edu/~math21a/` for the revised assignment schedule.

1. Computing partial derivatives

(1) Compute all first partial derivatives of the following functions:

(a)
$$f(x, y) = \sin x \cos y$$

(b)
$$f(x, y, z) = \ln(x + 2y + 3z)$$

(c)
$$f(x, y) = e^{\sin(x/y)}$$

2. Higher partial derivatives

(1) Compute f_{xy} and f_{yx}, where

$$f(x,y) = \ln\sqrt{x^2+y^2}$$

Does Clairaut's theorem hold?

(2) Can you find a function $f(x,y)$ such that

$$\begin{aligned} f_x &= x+4y \\ f_y &= 3x-y \end{aligned}$$

If not, why not?

3. Something harder

(1) We can interpret $f_x(a,b)$ as the derivative of $f(x,y)$ in the direction $\langle 1,0\rangle$ at the point (a,b). Similarly, we can interpret $f_y(a,b)$ as the derivative of $f(x,y)$ in the direction $\langle 0,1\rangle$ at the point (a,b). In this exercise, we will compute the derivative of $f(x,y)$ in *any* direction at the point (a,b).
   (a) Find an equation for the tangent plane to the graph $z = f(x,y)$ at the point $(a,b,f(a,b))$. (It may help to consult Figure 1 on page 769 of your textbook.)
   (b) Use this to compute the rate of change of $f(x,y)$ in the direction $\langle \alpha,\beta\rangle$.
   (c) Show that we can write the answer to (b) in the form

$$\nabla f \cdot \langle \alpha,\beta\rangle$$

   for some vector ∇f. This vector is called the *gradient* of the function $f(x,y)$.