FRIDAY 24TH OCTOBER : TANGENT PLANES AND LINEAR APPROXIMATIONS

Reading: sections 11.4 and 11.5
Homework: see `www.courses.fas.harvard.edu/~math21a/` for the revised assignment schedule.

1. Tangent planes and linear approximations

(1) Give an equation for the tangent plane to the surface $z = e^{x^2-y^2}$ at the point $(1, -1, 1)$.

(2) Compute the linearization of the function

$$f(x, y) = \ln(2x - y)$$

at the point $(1, 1)$. Use it to estimate $f(0.9, 1.05)$.

2. Differentiability

(1) Show that the function
$$f(x,y) = \frac{x}{y}$$
is differentiable at the point $(x,y) = (6,3)$ and compute the linearization of the function at that point.

(2) At what points are the following functions differentiable?
   (a) $f(x,y) = e^{xy}\cos(1+\pi xy)$
   (b) $f(x,y) = x - y\ln|x+y|$

3. A harder problem

(1) Prove that if $f(x,y)$ is differentiable at the point (a,b) then it is continuous at the point (a,b). Give an example which shows that the converse statement is false.
(2) Let
$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$
Show that $f_x(0,0)$ and $f_y(0,0)$ both exist (you will need to use the definition of partial derivatives as limits here).
(3) Is $f(x,y)$ differentiable at $(0,0)$?