SOLUTIONS: TANGENT PLANES...

SECTION ONE

1 Let $f(x,y) = e^{x^2-y^2}$. Then the tangent plane has equation

$$z - 1 = f_x(1,-1)(x-1) + f_y(1,-1)(y+1)$$

$f_x = 2x\,e^{x^2-y^2}$ and $f_y = -2y\,e^{x^2-y^2}$, so the tangent plane is

$$z - 1 = 2(x-1) + 2(y+1)$$

or $$2x + 2y - z = -1$$

2 The linearization is

$$g(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$$
$$= 0 + 2(x-1) - (y-1)$$
$$= 2x - y - 1$$

so $$f(0.9, 1.05) \approx g(0.9, 1.05)$$
$$= \underline{\underline{-0.25}}$$

SECTION TWO

(1) $f_x = 1/y$

$f_y = -x/y^2$

and these are both continuous near $(6,3)$ [the only discontinuities lie on the line $y=0$] so f is differentiable at $(6,3)$.

The linearization is

$$f(6,3) + f_x(6,3)(x-6) + f_y(6,3)(y-3)$$

$$= 2 + \tfrac{1}{3}(x-6) - \tfrac{2}{3}(y-3)$$

$$= \underline{\underline{2 + \tfrac{1}{3}x - \tfrac{2}{3}y}}$$

(2) (a) Everywhere (f is the product of compositions of everywhere-differentiable functions)

(b) Everywhere except possibly along the line $y=-x$, since $|x+y|$ is differentiable except on the line $y=x$, $\ln t$ is differentiable for all $t>0$, and the composition (and product) of differentiable functions is differentiable.

SECTION THREE

(1) Since f is differentiable at (a,b) we know that

$$f(a+h, b+k) = f(a,b) + h f_x(a,b) + k f_y(a,b) + h \varepsilon_1(h,k) + k \varepsilon_2(h,k)$$

where $\varepsilon_1(h,k)$ and $\varepsilon_2(h,k)$ tend to zero as $(h,k) \to 0$. Thus

$$\lim_{(h,k)\to(0,0)} f(a+h, b+k) = f(a,b)$$

and so f is continuous at (a,b).

$f(x,y) = |x|$ is continuous everywhere but not differentiable at $(0,0)$.

(2) $f_x(0,0) = \lim_{h\to 0} f(h,0) = \lim_{h\to 0} 0 = 0$

$f_y(0,0) = 0$ similarly

(3) Since we know from the worksheet on limits and continuity that $f(x,y)$ is not continuous at $(0,0)$, part (a) implies that it cannot be differentiable there.