MONDAY 27TH OCTOBER : DIFFERENTIABILITY / CHAIN RULE

Reading: sections 11.5 and 11.6
Homework: see `www.courses.fas.harvard.edu/~math21a/` for the revised assignment schedule.

1. Differentiability

(1) Show that
$$f(x, y) = \tan^{-1}(x + 2y)$$
is differentiable at the point $(1, 0)$. Estimate $f(1.1, -0.05)$.

2. Chain Rule

(1) Suppose that $z = \sin x \cos y$, where $x = \pi t$ and $y = \sqrt{t}$. Use the Chain Rule to compute $\frac{dz}{dt}$ at $t = 1$.

(2) Suppose that $z = x/y$, where $x = se^{-t}$ and $y = 1+se^{-t}$. Compute the partial derivatives of z with respect to s and to t.

(3) The radius of a right circular cone is increasing at a rate of 1.8 inches per second, and the height is decreasing at a rate of 2.5 inches per second. Right now the cone has radius 120 inches and height 180 inches. At what rate is the volume changing?

3. A harder problem : the wave equation

(1) (a) Show that any function of the form

$$z = f(x + ct) + g(x - ct)$$

is a solution to the *wave equation*

$$z_{tt} = c^2 z_{xx}$$

(You may want to make the change-of-variables $u = x + ct$, $v = x - ct$.)

(b) The function $z(x,t)$ represents the vertical displacement of a (taut) string at the point x and time t. The solutions $z = f(x + ct)$ are called *left-moving solutions*, and the solutions $z = g(x - ct)$ are called *right-moving solutions*. Why? What does c represent here?

(c) Express z_{tt} and z_{xx} in terms of derivatives with respect to u and v. Deduce that *any* solution to the wave equation has the form

$$z = f(x + ct) + g(x - ct)$$