WEDNESDAY 29TH OCTOBER : CHAIN RULE / DIRECTIONAL DERIVATIVES

Reading: sections 11.6 and 11.7
Homework: see `www.courses.fas.harvard.edu/~math21a/` for the revised assignment schedule.

1. Chain Rule

(1) Suppose that $f(x, y)$ is a function of two variables, and that (r, θ) are the usual polar co-ordinates in the xy-plane. Compute the partial derivatives f_r and f_θ in terms of f_x and f_y. Check your answer by computing f_r and f_θ when $f(x, y) = \sqrt{x^2 + y^2}$.

(2) The temperature in degrees Celcius at the point (x, y) is $T(x, y)$. A bug crawls so that its position after t seconds is

$$x = \sqrt{1+t} \qquad y = 2 + \frac{t}{3}$$

where x and y are measured in centimeters. Suppose that $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$. How fast is the temperature along the bug's path rising after 3 seconds?

(3) Car A is travelling north on one highway, Car B is travelling west on another highway. They are both approaching the intersection of the two highways. Right now, Car A is 2km from the intersection and going at 90km/h and Car B is 1km from the intersection and going at 70km/h. How fast is the distance between the cars changing?

2. Directional derivatives

(1) Find the directional derivative of

$$f(x, y) = \sin(x + 2y)$$

at $(4, 2)$ in the direction which makes an angle $\theta = \frac{3\pi}{4}$ with the positive x-axis. Find the direction in which the function increases most steeply.

3. A harder problem

(1) Show that the gradient vector

$$\nabla f = \langle f_x, f_y \rangle$$

at the point (a, b) points perpendicular to the level curve of f which passes through (a, b).