FRIDAY 3RD OCTOBER : SURFACES

Reading: sections 9.6 and 9.7
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Sketching surfaces using traces

(1) Consider the surface in 3D space given by the equation

$$x^2 + y^2 - z^2 = 1$$

(a) Draw the traces of this surface in the horizontal planes $z = k$ for $k = -2, -1, 0, 1, 2$.

(b) Draw the traces of this surface in the vertical planes $x = k$ for $k = -1, 0, 1, 2$.

(c) Draw the traces of this surface in the vertical planes $y = k$ for $k = -1, 0, 1, 2$.

(d) Sketch the surface.

2. A more challenging problem

(1) One of the ways that we have seen to describe the graph of a function $f(x, y)$ of two variables is to sketch *level curves*

$$f(x, y) = k$$

for various values of the constant k. These are curves in 2D space. We can use the same idea when thinking about a function $f(x, y, z)$ of three variables, except that this time we get *level surfaces*

$$f(x, y, z) = k$$

in 3D space.

Sketch some level surfaces for the following functions

(a) $f(x, y, z) = 3x + 4y + 12z$

(b) $f(x, y, z) = x^2 + y^2 + z^2$

Use your answer to show that the maximum value of

$$g(x, y, z) = 3x + 4y + 12z$$

on the sphere

$$S = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$$

occurs at $(\frac{3}{13}, \frac{4}{13}, \frac{12}{13})$. Where is the minimum value?

Show that the maximum value of the function

$$h(x, y, z) = 4z^2 - x^2 - 2y^2$$

on the sphere S is 4. What is the minimum value?