FRIDAY 31ST OCTOBER : DIRECTIONAL DERIVATIVES AND THE GRADIENT VECTOR

Reading: sections 11.6 and 11.7
Homework: see `www.courses.fas.harvard.edu/~math21a/` for the revised assignment schedule.

1. Directional Derivatives

(1) Find the rate of change of

$$f(x, y) = e^y \cos x$$

at the point $(\frac{\pi}{4}, 0)$ in the direction of $\mathbf{i} + 3\mathbf{j}$.

(2) Find the rate of change of

$$f(x, y, z) = xy^2z^3$$

at the point $(3, 2, 1)$ in the direction of $2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

2. The gradient vector

(1) Find the maximum rate of increase of the function

$$f(x, y, z) = x + \frac{y}{z}$$

at the point $(4, 3, -1)$. In which direction does it occur?

(2) The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at $(1, 2, 2)$ is 120 degrees Celcius.
   (a) Find the rate of change of T at $(1, 2, 2)$ in the direction towards $(2, 1, 3)$.
   (b) Show that at any point in the ball, the direction of greatest increase in temperature points towards the center of the ball.

3. Tangent planes and normal lines to surfaces

(1) Find an equation for the tangent plane to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 2$$

at $(2, 0, 3)$.

(2) Find an equation for the tangent plane to the surface

$$z - f(x, y) = 0$$

at the point $(a, b, f(a, b))$.

(3) Where does the tangent plane to the hyperboloid

$$x^2 + y^2 - z^2 = 1$$

at $(1, 1, -1)$ meet the normal line to

$$z = 4 - x^2 - y^2$$

at $(2, 2, -4)$?