MONDAY 6TH OCTOBER : CYLINDRICAL AND SPHERICAL CO-ORDINATES

Reading: sections 9.7 and 10.1
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Cylindrical Co-ordinates

(1) What are the equations of the following surfaces in Cartesian co-ordinates? Sketch the surfaces.
  (a) $z = r$
  (b) $z = r^2$

(2) Consider the curve where the surfaces

$$z = x^2 + y^2 \qquad \text{and} \qquad z = x$$

meet.
  (a) Write down formulas in cylindrical polar co-ordinates that describe the curve. What does the curve look like when projected to the xy-plane?

  (b) Show that a parametrization of the curve is given by

$$x = \cos^2\theta \quad y = \cos\theta\sin\theta \quad z = \cos^2\theta \qquad 0 \le \theta \le \pi$$

2. Spherical Co-ordinates

(1) Sketch the surface given by

$$\rho = 1 \qquad 0 \leq \phi \leq \frac{\pi}{4}$$

Describe it in words.

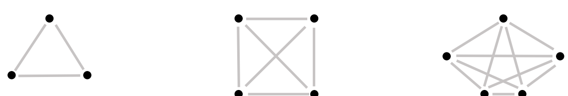
(2) Sketch the surface given by

$$\rho \cos \phi = \rho^2 \sin^2 \phi \cos 2\theta$$

3. A challenging problem : Ramsey numbers

This has nothing to do with the material of today's class.

- First, show that any collection of 6 people contains either three people who all know each other or three people none of whom know each other. One way to start is to rephrase the problem in terms of *graph theory*. A graph consists of *vertices*, which we draw as dots, some of which are joined by *edges* which we draw as lines. For example, the *complete graph on n vertices* $K(n)$ consists of all possible edges between n vertices. Here is a picture of $K(3)$, $K(4)$ and $K(5)$. You should imagine that the edges do not meet, but pass over or under each other.



K(3) K(4) K(5)

In this language, our problem becomes to show that if we take the graph $K(6)$ and color each edge either red or blue then *however we color the graph* we can either find a subgraph which is an all-red $K(3)$ or a subgraph which is an all-blue $K(3)$. First figure out why this is the same as our original problem, then solve it.

- Now show that if we color each edge of a $K(10)$ either red or blue then however we color the graph we can either find a subgraph which is an all-red $K(4)$ or a subgraph which is an all-blue $K(3)$.
Define the *Ramsey number* $R(m, n)$ to be the least number N such that if we color each edge of a $K(N)$ either red or blue then however we color the graph we can either find a subgraph which is an all-red $K(m)$ or a subgraph which is an all-blue $K(n)$.
- Show that $R(m, n) = R(n, m)$.
- Show that, for each m and n, the Ramsey number $R(m, n)$ is finite.
- What happens if we use more than two colors?
- Look up Ramsey numbers on MathWorld (`http://mathworld.wolfram.com`)