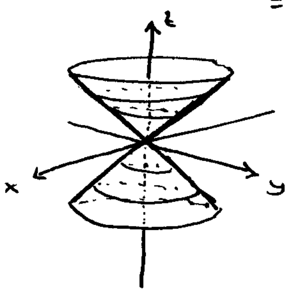
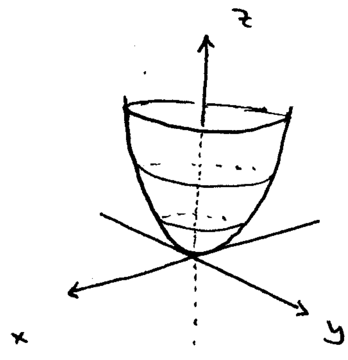
SOLUTIONS: CYLINDRICAL AND SPHERICAL CO-ORDINATES

SECTION ONE

1 (a) $z^2 = r^2$
$= x^2 + y^2$ so $z = \pm\sqrt{x^2+y^2}$



1 (b) $z = x^2 + y^2$

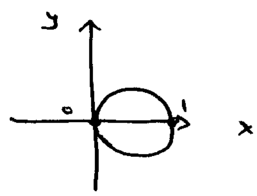


(2) (a) They meet at $\begin{cases} z = x^2+y^2 \\ z = x \end{cases}$

i.e. at $\begin{cases} z = r\cos\theta \\ z = r^2 \end{cases}$

To project to the xy-plane, eliminate z: $r^2 = r\cos\theta$

This is a circle (it's $x^2+y^2 = x \Rightarrow (x-\frac{1}{2})^2 + y^2 = \frac{1}{4}$)



2 (b) We need to show that

(i) each point $x = \cos^2\theta$, $y = \cos\theta \sin\theta$, $z = \cos^2\theta$, $0 \leq \theta \leq \pi$ lies on the curve

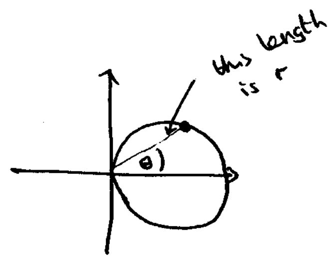
(ii) and that each point on the curve can be written in that form.

For (i), plug in:

$$\left\{ \begin{array}{lll} z = x^2 + y^2 & \rightsquigarrow & \cos^2\theta = \cos^2\theta(\cos^2\theta + \sin^2\theta) \\ & & \text{which is true} \\ z = x & \rightsquigarrow & \cos^2\theta = \cos^2\theta \end{array} \right.$$

these are the equations defining the curve

For (ii), let θ be this angle:



Then, since $r = \cos\theta$ we have

$$x = r\cos\theta$$
$$= \cos^2\theta$$

$$y = r\sin\theta = \cos\theta \sin\theta$$

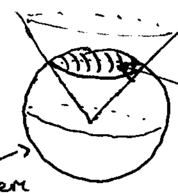
and $z = x = \cos^2\theta$

The curve is swept out once as θ runs from 0 to π.

SOLUTIONS: CYLINDRICAL AND SPHERICAL CO-ORDS

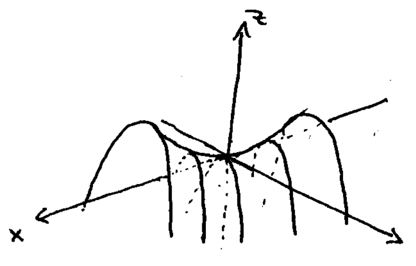
SECTION 2

(1) This is the "top portion" of the unit sphere



(2)
$$\rho \cos\phi = \rho^2 \sin^2\phi \cos 2\theta$$
$$= \rho^2 \sin^2\phi \,(\cos^2\theta - \sin^2\theta)$$

so $z = x^2 - y^2$ which is a saddle (sketch slices!)



SECTION 3

(a) Take one vertex for each person. Colour the edge from person P to person Q red if they know each other and blue if they don't.

Pick a vertex of the graph. This has 5 edges coming out of it, so either 3 of them are blue or 3 are red. Without loss of generality, 3 are red.

Suppose the graph contains no red $K(3)$. Then every edge between the labelled edges must be blue. Thus the graph contains a blue $K(3)$!

- $R(m,n) = R(n,m)$ by symmetry (flip red and blue!)
- The above argument, slightly generalized, shows that if $N = R(m-1,n) + R(m,n-1)$ then any red-blue coloured $K(N)$ contains a red $K(m)$ or blue $K(n)$.

By induction, therefore, $R(m,n)$ is finite for all m and n.

A similar argument works for three or more colours.