WEDNESDAY 8TH OCTOBER : SPACE CURVES

Reading: sections 10.1 and 10.2
Homework: see `www.courses.fas.harvard.edu/~math21a/`
note that the first computing assignment is due next Wednesday

1. Sketching space curves

(1) Sketch the curves given by
   (a) $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j}$
   (b) $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$

(2) Sketch the curve with vector equation

$$\mathbf{r}(t) = \left\langle \sin t, \cos t, t^2 \right\rangle$$

(3) Sketch the curve with vector equation

$$\mathbf{r}(t) = \left\langle t\cos t, t\sin t, t^2 \right\rangle$$

(Hint: $z = x^2 + y^2$!)

2. Describing space curves

(1) Sketch the following curves, and find parametrizations for them:
   (a) The intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$

   (b) The intersection of the surfaces in 3D space with equations $z = x^2 + 3y^2$ and $x = y^2$.

3. A challenging problem : arc length

(1) Suppose that a curve C has parametrization

$$\mathbf{r}(t) \qquad a \leq t \leq b$$

Explain why the length of the curve C is given by

$$L = \int_a^b \|\mathbf{r}'(t)\| \, dt$$

Show also that this quantity is independent of the choice of parametrization. In other words, show that two different parametrizations $\mathbf{r}_1(t)$, $a_1 \leq t \leq b_1$ and $\mathbf{r}_2(t)$, $a_2 \leq t \leq b_2$ of C give the same value for L.