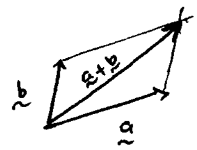
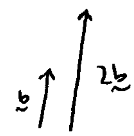
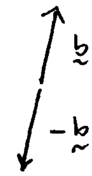
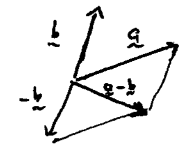
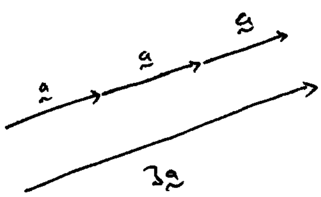
WORKSHEET SOLUTIONS : VECTORS





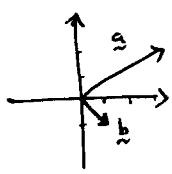






$\underset{\sim}{a} + \underset{\sim}{a} + \underset{\sim}{a} = 3\underset{\sim}{a}$

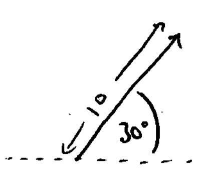
$\underset{\sim}{a} = \langle 3, 2 \rangle$ $\quad$ $\underset{\sim}{b} = \langle 1, -1 \rangle$



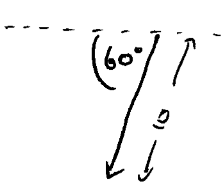
$$\underset{\sim}{a} + \underset{\sim}{b} = \langle 4, 1 \rangle$$

$$\underset{\sim}{a} - \underset{\sim}{b} = \langle 2, 3 \rangle$$

$$5\underset{\sim}{b} = \langle 5, -5 \rangle$$



This vector is $\langle 10 \cos 30°, 10 \sin 30° \rangle = \langle 5\sqrt{3}, 5 \rangle$



This is $\langle -10 \cos 60°, -10 \sin 60° \rangle = \langle -5, -5\sqrt{3} \rangle$

A unit vector in the NW direction is $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

so the velocity of the train is $\langle -\frac{30}{\sqrt{2}}, \frac{30}{\sqrt{2}} \rangle$

The velocity of the marble (relative to the train) is $\langle 0, -2 \rangle$

and so the velocity of the marble (relative to you) is

$$\left\langle -\frac{30}{\sqrt{2}}, \frac{30}{\sqrt{2}} \right\rangle + \langle 0, -2 \rangle = \left\langle -\frac{30}{\sqrt{2}}, \frac{30}{\sqrt{2}} - 2 \right\rangle$$

The speed of the marble (relative to you) is therefore

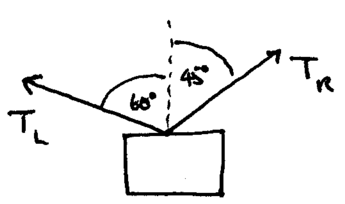
$$\left\| \left\langle -\frac{30}{\sqrt{2}}, \frac{30}{\sqrt{2}} - 2 \right\rangle \right\| = 2\sqrt{226 - 15\sqrt{2}}$$

$$\approx \underline{\underline{28.62 \ ms^{-1}}}$$

Let the tension in the left-hand rope be $\underset{\sim}{T_L}$ and have magnitude L.

Let the tension in the right-hand rope be $\underset{\sim}{T_R}$ and have magnitude R.

$$T_L = \langle -L \sin 60°, L \cos 60° \rangle = \left\langle -\frac{\sqrt{3}L}{2}, \frac{L}{2} \right\rangle$$



$$T_R = \langle R \sin 45°, R \cos 45° \rangle = \left\langle \frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}} \right\rangle$$

WORKSHEET SOLUTION : VECTORS

We know that $T_L + T_R = \langle 0, 100 \rangle$ (taking $g = 10$)

$$\Rightarrow \left\langle \frac{R}{\sqrt{2}} - \frac{\sqrt{3}L}{2}, \frac{R}{\sqrt{2}} + \frac{L}{2} \right\rangle = \langle 0, 100 \rangle$$

Solving
$$\begin{cases} \dfrac{R}{\sqrt{2}} - \dfrac{\sqrt{3}L}{2} = 0 \\[2ex] \dfrac{R}{\sqrt{2}} + \dfrac{L}{2} = 0 \end{cases}$$

we find

$$\boxed{\begin{aligned} R &= \frac{100\sqrt{6}}{1+\sqrt{3}} \approx 89.66\ \text{N} \\[2ex] L &= \frac{200}{1+\sqrt{3}} \approx 73.21\text{N} \end{aligned}}$$