FRIDAY 26TH SEPTEMBER : DOT PRODUCT

Reading: sections 9.3 and 9.4
Homework: 9.3 Q8, 16, 18, 25, 33, 34, 37
9.4 Q8, 9, 16, 18, 22, 27

1. WORK, LENGTH, ANGLE

(1) Let
$$\mathbf{a} = \langle 5, 1, 3 \rangle \qquad \mathbf{b} = \langle 1, 0, -2 \rangle$$
Use the dot product to find:
 (a) The length of $\mathbf{a}$.
 (b) The length of $\mathbf{b}$.
 (c) The angle between $\mathbf{a}$ and $\mathbf{b}$.

(2) A railroad runs horizontally in a straight line from the point $(0, 0)$ to the point $(10, 10)$. A railcar full of coal, with mass 500kg, is sitting at $(0, 0)$. A donkey pulls the railcar to the point $(10, 10)$ using a force of 200N in the direction of $\langle 2, 1 \rangle$. How much work does the donkey do?

(3) Find a unit vector in the direction of
 (a) $\langle 3, -5, 1 \rangle$
 (b) $\mathbf{x}$ (Your answer should involve the dot product)

2. Scalar and Vector Projections

(1) Find the scalar projection of **b** onto **a** for
   (a) $\mathbf{a} = \langle 1, 1 \rangle$, $\mathbf{b} = \langle 2, 3 \rangle$.
   (b) $\mathbf{a} = \langle 1, -1, 4 \rangle$, $\mathbf{b} = \langle 2, 0, 1 \rangle$.

(2) Find the vector projection of **b** onto **a** for
   (a) $\mathbf{a} = \langle 1, 1 \rangle$, $\mathbf{b} = \langle 2, 3 \rangle$.
   (b) $\mathbf{a} = \langle 1, -1, 4 \rangle$, $\mathbf{b} = \langle 2, 0, 1 \rangle$.

3. Two useful inequalities

(1) Show that
$$|\mathbf{a}.\mathbf{b}| \leq |\mathbf{a}||\mathbf{b}|$$
This is called the *Cauchy-Schwartz inequality*. Deduce that
$$|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$$
This is called the *triangle inequality*. To see why, draw a picture!