SOLUTIONS : DOT PRODUCT

SECTION 1

1 (a) $|\underset{\sim}{a}| = \sqrt{\underset{\sim}{a}\cdot\underset{\sim}{a}} = \sqrt{25+1+9} = \sqrt{35}$

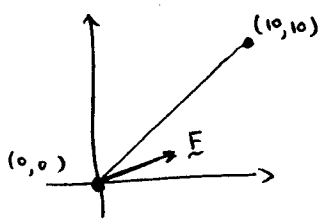
(b) $|\underset{\sim}{b}| = \sqrt{5}$

(c) $\underset{\sim}{a}\cdot\underset{\sim}{b} = 5 - 6 = -1$

so $|\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta = -1$

$\Rightarrow \cos\theta = \frac{-1}{5\sqrt{7}}$ and so $\theta = \cos^{-1}\left(-\frac{1}{5\sqrt{7}}\right)$

2



Unit vector in the direction $\langle 2,1\rangle$ is $\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$

so $\underset{\sim}{F} = \left\langle \frac{400}{\sqrt{5}}, \frac{200}{\sqrt{5}} \right\rangle$

and Work done $= \underset{\sim}{F}\cdot\underset{\sim}{D}$

$= \left\langle \frac{400}{\sqrt{5}}, \frac{200}{\sqrt{5}} \right\rangle \cdot \langle 10, 10\rangle$

$= \frac{6000}{\sqrt{5}} \approx 2680 \text{ J}$

3 (a) $\langle 3,-5,1\rangle \cdot \langle 3,-5,1\rangle = 35$, so

unit vector $= \left\langle \frac{3}{\sqrt{35}}, \frac{-5}{\sqrt{35}}, \frac{1}{\sqrt{35}} \right\rangle$

(b) $\frac{\underset{\sim}{x}}{|\underset{\sim}{x}|} = \frac{\underset{\sim}{x}}{\sqrt{\underset{\sim}{x}\cdot\underset{\sim}{x}}}$

SECTION 2

1 (a) $\text{comp}_{\underset{\sim}{a}} \underset{\sim}{b} = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|} = \dfrac{5}{\sqrt{2}}$

(b) $\text{comp}_{\underset{\sim}{a}} \underset{\sim}{b} = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|} = \dfrac{6}{\sqrt{18}} = \sqrt{2}$

2 (a) $\text{proj}_{\underset{\sim}{a}} \underset{\sim}{b} = \left(\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|^2}\right) \underset{\sim}{a} = \dfrac{5}{2}\langle 1, 1\rangle = \left\langle \dfrac{5}{2}, \dfrac{5}{2} \right\rangle$

(b) $\text{proj}_{\underset{\sim}{a}} \underset{\sim}{b} = \left(\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|^2}\right) \underset{\sim}{a} = \dfrac{1}{3}\langle 1, -1, 4\rangle = \left\langle 1/3, -1/3, 4/3 \right\rangle$

SECTION 3

1 $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$

so $|\underset{\sim}{a} \cdot \underset{\sim}{b}| = |\underset{\sim}{a}||\underset{\sim}{b}||\cos\theta|$. Since $-1 \leq \cos\theta \leq 1$, we have $|\underset{\sim}{a} \cdot \underset{\sim}{b}| \leq |\underset{\sim}{a}||\underset{\sim}{b}|$.

For the second part, $|\underset{\sim}{a} + \underset{\sim}{b}|^2 = (\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b})$

$= \underset{\sim}{a} \cdot \underset{\sim}{a} + 2\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{b}$

$\leq |\underset{\sim}{a}|^2 + 2|\underset{\sim}{a}||\underset{\sim}{b}| + |\underset{\sim}{b}|^2$ (using Cauchy-Schwarz)

$= (|\underset{\sim}{a}| + |\underset{\sim}{b}|)^2$

Since both quantities being squared are positive, $|\underset{\sim}{a} + \underset{\sim}{b}| \leq |\underset{\sim}{a}| + |\underset{\sim}{b}|$.