SOLUTIONS: CROSS PRODUCT

SECTION 1

1 (a) $\underset{\sim}{a} \times \underset{\sim}{b} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 2 & 0 & -1 \\ 0 & 1 & 1 \end{vmatrix} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$

$\underset{\sim}{b} \times \underset{\sim}{a} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 0 & 1 & 1 \\ 2 & 0 & -1 \end{vmatrix} = -\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$

2 Suppose that the diagram is in the xy-plane. Then the force $\underset{\sim}{F}$ applied is $\langle -36\cos 30°, -36\sin 30°, 0 \rangle$

i.e. $\underset{\sim}{F} = \langle -18\sqrt{3}, -18, 0 \rangle$

It is applied at the point $\underset{\sim}{r} = \langle 4, -4, 0 \rangle$

so torque $= \underset{\sim}{r} \times \underset{\sim}{F} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 4 & -4 & 0 \\ -18\sqrt{3} & -18 & 0 \end{vmatrix} = 72(\sqrt{3}-1)\underset{\sim}{k}$

Thus the magnitude of the torque is $72(\sqrt{3}-1) \approx 52.7$ lb ft

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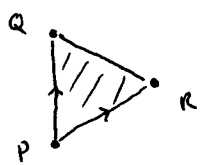
SECTION 2

(1)



$$\text{area} = \tfrac{1}{2}\,|\overrightarrow{PQ} \times \overrightarrow{PR}|$$

but $\overrightarrow{PQ} = \langle -1, 1, 0 \rangle$

$\overrightarrow{PR} = \langle -1, 0, 1 \rangle$

so $$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \langle 1, 1, 1 \rangle$$

and $$\text{area} = \tfrac{1}{2}\,|\langle 1,1,1 \rangle| = \frac{\sqrt{3}}{2}$$

$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle 1,1,1 \rangle$ is a vector perpendicular to the plane through P, Q and R.

(2) Volume $= \underline{a} \cdot (\underline{b} \times \underline{c})$

$$= (3\underline{i} - \underline{k}) \cdot \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 0 \\ 4 & 1 & 1 \end{vmatrix}$$

$$= (3\underline{i} - \underline{k}) \cdot (\underline{i} - \underline{j} - 3\underline{k})$$

$$= 6$$