**7.** True. If f has a local minimum and f is differentiable at (a, b) then by Theorem 11.7.2, $f_x(a, b) = 0$ and $f_y(a, b) = 0$, so $\nabla f(a, b) = \langle f_x(a, b), f_y(a, b) \rangle = \langle 0, 0 \rangle = \mathbf{0}$.

8. False. The limit does not exist because the function is not defined on the line $y = x$, and so we have a path approaching the point $(1, 1)$ along which f does not approach $\frac{1}{2}$.

9. False. $\nabla f(x, y) = \langle 0, 1/y \rangle$.

10. True. This is part (c) of the Second Derivatives Test (11.7.3).

11. True. $\nabla f = \langle \cos x, \cos y \rangle$, so $|\nabla f| = \sqrt{\cos^2 x + \cos^2 y}$. But $|\cos \theta| \le 1$, so $|\nabla f| \le \sqrt{2}$. Now $D_{\mathbf{u}} f(x, y) = \nabla f \cdot \mathbf{u} = |\nabla f| \, |\mathbf{u}| \cos \theta$, but $\mathbf{u}$ is a unit vector, so $|D_{\mathbf{u}} f(x, y)| \le \sqrt{2} \cdot 1 \cdot 1 = \sqrt{2}$.

12. False. See Exercise 11.7.29.
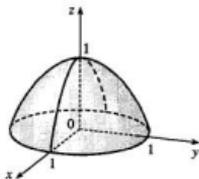
◆ **EXERCISES** ◆

1. The domain of $\sin^{-1} x$ is $-1 \le x \le 1$ while the domain of $\tan^{-1} y$ is all real numbers, so the domain of $f(x, y) = \sin^{-1} x + \tan^{-1} y$ is $\{(x, y) \mid -1 \le x \le 1\}$.
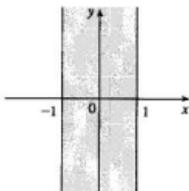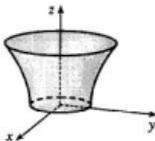

2. $D = \{(x, y, z) \mid z \ge x^2 + y^2\}$, the points on and above the paraboloid $z = x^2 + y^2$.
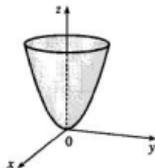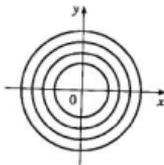

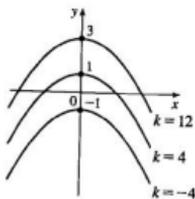**3.** $z = f(x, y) = 1 - x^2 - y^2$, a paraboloid with vertex $(0, 0, 1)$.

4. $z = f(x, y) = \sqrt{x^2 + y^2 - 1}$, so $z \ge 0$ and $1 = x^2 + y^2 - z^2$. Thus the graph is the upper half of a hyperboloid of one sheet.
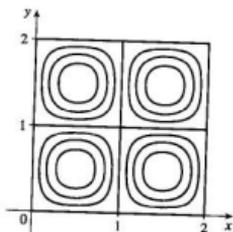
5. Let $k = e^{-c} = e^{-(x^2+y^2)}$ be the level curves. Then
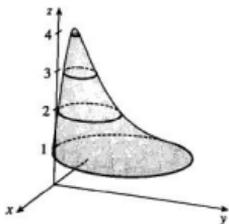$-\ln k = c = x^2 + y^2$, so we have a family of
concentric circles.

6. $k = x^2 + 4y$ or $4(y - k/4) = -x^2$, a family of
parabolas with vertex at $(0, k/4)$.

7.

8.

9. f is a rational function, so it is continuous on its domain. Since f is defined at $(1, 1)$, we use direct substitution to

evaluate the limit: $\lim\limits_{(x,y)\to(1,1)} \dfrac{2xy}{x^2 + 2y^2} = \dfrac{2(1)(1)}{1^2 + 2(1)^2} = \dfrac{2}{3}$.

10. As $(x, y) \to (0, 0)$ along the x-axis, $f(x, 0) = 0/x^2 = 0$ for $x \neq 0$, so $f(x, y) \to 0$ along this line. But
$f(x, x) = 2x^2/(3x^2) = \frac{2}{3}$, so as $(x, y) \to (0, 0)$ along the line $x = y$, $f(x, y) \to \frac{2}{3}$. Thus the limit doesn't exist.

11. (a) $T_x(6, 4) = \lim\limits_{h\to 0} \dfrac{T(6 + h, 4) - T(6, 4)}{h}$, so we can approximate $T_x(6, 4)$ by considering $h = \pm 2$ and using the

values given in the table: $T_x(6, 4) \approx \dfrac{T(8, 4) - T(6, 4)}{2} = \dfrac{86 - 80}{2} = 3$,

$T_x(6, 4) \approx \dfrac{T(4, 4) - T(6, 4)}{-2} = \dfrac{72 - 80}{-2} = 4$. Averaging these values, we estimate $T_x(6, 4)$ to be

approximately $3.5\,^\circ\text{C}\,/\text{m}$. Similarly, $T_y(6, 4) = \lim\limits_{h\to 0} \dfrac{T(6, 4 + h) - T(6, 4)}{h}$, which we can

approximate with $h = \pm 2$: $T_y(6, 4) \approx \dfrac{T(6, 6) - T(6, 4)}{2} = \dfrac{75 - 80}{2} = -2.5$,

$T_y(6, 4) \approx \dfrac{T(6, 2) - T(6, 4)}{-2} = \dfrac{87 - 80}{-2} = -3.5$. Averaging these values, we estimate $T_y(6, 4)$ to be

approximately $-3.0\,^\circ\text{C}\,/\text{m}$.

(b) Here $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$, so by Equation 11.6.9, $D_{\mathbf{u}}T(6,4) = \nabla T(6,4) \cdot \mathbf{u} = T_x(6,4) \frac{1}{\sqrt{2}} + T_y(6,4) \frac{1}{\sqrt{2}}$. Using

our estimates from part (a), we have $D_{\mathbf{u}}T(6,4) \approx (3.5) \frac{1}{\sqrt{2}} + (-3.0) \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \approx 0.35$. This means that as

we move through the point $(6,4)$ in the direction of $\mathbf{u}$, the temperature increases at a rate of approximately

$0.35\,°\text{C}\,/\text{m}$.

Alternatively, we can use Definition 11.6.2: $D_{\mathbf{u}}T(6,4) = \lim\limits_{h \to 0} \dfrac{T\left(6 + h\frac{1}{\sqrt{2}}, 4 + h\frac{1}{\sqrt{2}}\right) - T(6,4)}{h}$, which we

can estimate with $h = \pm 2\sqrt{2}$. Then $D_{\mathbf{u}}T(6,4) \approx \dfrac{T(8,6) - T(6,4)}{2\sqrt{2}} = \dfrac{80 - 80}{2\sqrt{2}} = 0$,

$D_{\mathbf{u}}T(6,4) \approx \dfrac{T(4,2) - T(6,4)}{-2\sqrt{2}} = \dfrac{74 - 80}{-2\sqrt{2}} = \dfrac{3}{\sqrt{2}}$. Averaging these values, we have

$D_{\mathbf{u}}T(6,4) \approx \dfrac{3}{2\sqrt{2}} \approx 1.1\,°\text{C}\,/\text{m}$.

(c) $T_{xy}(x,y) = \dfrac{\partial}{\partial y}[T_x(x,y)] = \lim\limits_{h \to 0} \dfrac{T_x(x,y+h) - T_x(x,y)}{h}$, so $T_{xy}(6,4) = \lim\limits_{h \to 0} \dfrac{T_x(6,4+h) - T_x(6,4)}{h}$

which we can estimate with $h = \pm 2$. We have $T_x(6,4) \approx 3.5$ from part (a), but we will also need values for

$T_x(6,6)$ and $T_x(6,2)$. If we use $h = \pm 2$ and the values given in the table, we have

$T_x(6,6) \approx \dfrac{T(8,6) - T(6,6)}{2} = \dfrac{80 - 75}{2} = 2.5$, $T_x(6,6) \approx \dfrac{T(4,6) - T(6,6)}{-2} = \dfrac{68 - 75}{-2} = 3.5$.

Averaging these values, we estimate $T_x(6,6) \approx 3.0$. Similarly,

$T_x(6,2) \approx \dfrac{T(8,2) - T(6,2)}{2} = \dfrac{90 - 87}{2} = 1.5$, $T_x(6,2) \approx \dfrac{T(4,2) - T(6,2)}{-2} = \dfrac{74 - 87}{-2} = 6.5$.

Averaging these values, we estimate $T_{xx}(6,2) \approx 4.0$. Finally, we estimate $T_{xy}(6,4)$:

$T_{xy}(6,4) \approx \dfrac{T_x(6,6) - T_x(6,4)}{2} = \dfrac{3.0 - 3.5}{2} = -0.25$,

$T_{xy}(6,4) \approx \dfrac{T_{xx}(6,2) - T_x(6,4)}{-2} = \dfrac{4.0 - 3.5}{-2} = -0.25$. Averaging these values, we have

$T_{xy}(6,4) \approx -0.25$.

12. From the table, $T(6,4) = 80$, and from Exercise 11 we estimated $T_x(6,4) \approx 3.5$ and $T_y(6,4) \approx -3.0$. The linear

approximation then is

$T(x,y) \approx T(6,4) + T_x(6,4)(x-6) + T_y(6,4)(y-4) \approx 80 + 3.5(x-6) - 3(y-4)$

$= 3.5x - 3y + 71$

Thus at the point $(5, 3.8)$, we can use the linear approximation to estimate

$T(5, 3.8) \approx 3.5(5) - 3(3.8) + 71 \approx 77.1\,°\text{C}$.

13. $f(x,y) = \sqrt{2x + y^2}$ $\Rightarrow$ $f_x = \frac{1}{2}(2x + y^2)^{-1/2}(2) = \dfrac{1}{\sqrt{2x + y^2}}$,

$f_y = \frac{1}{2}(2x + y^2)^{-1/2}(2y) = \dfrac{y}{\sqrt{2x + y^2}}$

14. $u = e^{-r} \sin 2\theta$ $\Rightarrow$ $u_r = -e^{-r} \sin 2\theta$, $u_\theta = 2e^{-r} \cos 2\theta$

15. $g(u,v) = u \tan^{-1} v$ $\Rightarrow$ $g_u = \tan^{-1} v$, $g_v = \dfrac{u}{1 + v^2}$

16. $w = \dfrac{x}{y-z}$ $\Rightarrow$ $w_x = \dfrac{1}{y-z}$, $w_y = x(-1)(y-z)^{-2} = -\dfrac{x}{(y-z)^2}$,

$w_z = x(-1)(y-z)^{-2}(-1) = \dfrac{x}{(y-z)^2}$

17. $T(p,q,r) = p\ln(q+e^r)$ $\Rightarrow$ $T_p = \ln(q+e^r)$, $T_q = \dfrac{p}{q+e^r}$, $T_r = \dfrac{pe^r}{q+e^r}$

18. $C = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.01T)(S - 35) + 0.016D$ $\Rightarrow$
$\partial C/\partial T = 4.6 - 0.11T + 0.00087T^2 - 0.01(S - 35)$, $\partial C/\partial S = 1.34 - 0.01T$, and $\partial C/\partial D = 0.016$. When
$T = 10$, $S = 35$, and $D = 100$ we have $\partial C/\partial T = 4.6 - 0.11(10) + 0.00087(10)^2 - 0.01(35 - 35) \approx 3.587$,
thus in $10\,^{\circ}\text{C}$ water with salinity 35 parts per thousand and a depth of 100 m, the speed of sound increases by about
3.59 m/s for every degree Celsius that the water temperature rises. Similarly, $\partial C/\partial S = 1.34 - 0.01(10) = 1.24$,
so the speed of sound increases by about 1.24 m/s for every part per thousand that the salinity of the water increases.
$\partial C/\partial D = 0.016$, so the speed of sound increases by about 0.016 m/s for every meter that the depth is increased.

19. $f(x,y) = 4x^3 - xy^2$ $\Rightarrow$ $f_x = 12x^2 - y^2$, $f_y = -2xy$, $f_{xx} = 24x$, $f_{yy} = -2x$, and $f_{xy} = f_{yx} = -2y$.

20. $z = xe^{-2y}$ $\Rightarrow$ $z_x = e^{-2y}$, $z_y = -2xe^{-2y}$, $z_{xx} = 0$, $z_{yy} = 4xe^{-2y}$, and $z_{xy} = z_{yx} = -2e^{-2y}$.

21. $f(x,y,z) = x^k y^l z^m$ $\Rightarrow$ $f_x = kx^{k-1}y^l z^m$, $f_y = lx^k y^{l-1} z^m$, $f_z = mx^k y^l z^{m-1}$,
$f_{xx} = k(k-1)x^{k-2}y^l z^m$, $f_{yy} = l(l-1)x^k y^{l-2}z^m$, $f_{zz} = m(m-1)x^k y^l z^{m-2}$, $f_{xy} = f_{yx} = klx^{k-1}y^{l-1}z^m$,
$f_{xz} = f_{zx} = kmx^{k-1}y^l z^{m-1}$, and $f_{yz} = f_{zy} = lmx^k y^{l-1}z^{m-1}$.

22. $v = r\cos(s + 2t)$ $\Rightarrow$ $v_r = \cos(s + 2t)$, $v_s = -r\sin(s + 2t)$, $v_t = -2r\sin(s + 2t)$, $v_{rr} = 0$,
$v_{ss} = -r\cos(s + 2t)$, $v_{tt} = -4r\cos(s + 2t)$, $v_{rs} = v_{sr} = -\sin(s + 2t)$, $v_{rt} = v_{tr} = -2\sin(s + 2t)$, and
$v_{st} = v_{ts} = -2r\cos(s + 2t)$.

23. $u = x^y$ $\Rightarrow$ $u_x = yx^{y-1}$, $u_y = x^y \ln x$ and $(x/y)u_x + (\ln x)^{-1}u_y = x^y + x^y = 2u$.

24. $\rho = \sqrt{x^2 + y^2 + z^2}$ $\Rightarrow$ $\rho_x = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}$, $\rho_{xx} = \dfrac{y^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}}$.

By symmetry, $\rho_{yy} = \dfrac{x^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}}$ and $\rho_{zz} = \dfrac{x^2 + y^2}{(x^2 + y^2 + z^2)^{3/2}}$. Thus

$\rho_{xx} + \rho_{yy} + \rho_{zz} = 2\dfrac{x^2 + y^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}} = \dfrac{2}{(x^2 + y^2 + z^2)^{1/2}} = \dfrac{2}{\rho}$.

25. (a) $z_x = 6x + 2$ $\Rightarrow$ $z_x(1,-2) = 8$ and $z_y = -2y$ $\Rightarrow$ $z_y(1,-2) = 4$, so an equation of the tangent plane is
$z - 1 = 8(x-1) + 4(y+2)$ or $z = 8x + 4y + 1$.

(b) A normal vector to the tangent plane (and the surface) at $(1,-2,1)$ is $(8,4,-1)$. Then parametric equations for
the normal line there are $x = 1 + 8t$, $y = -2 + 4t$, $z = 1 - t$, and symmetric equations are
$\dfrac{x-1}{8} = \dfrac{y+2}{4} = \dfrac{z-1}{-1}$.

26. (a) $z_x = e^x \cos y$ $\Rightarrow$ $z_x(0,0) = 1$ and $z_y = -e^x \sin y$ $\Rightarrow$ $z_y(0,0) = 0$, so an equation of the tangent plane
is $z - 1 = 1(x - 0) + 0(y - 0)$ or $z = x + 1$.

(b) A normal vector to the tangent plane (and the surface) at $(0,0,1)$ is $(1,0,-1)$. Then parametric equations for
the normal line there are $x = t$, $y = 0$, $z = 1 - t$, and symmetric equations are $x = 1 - z$, $y = 0$.

27. (a) Let $F(x, y, z) = x^2 + 2y^2 - 3z^2$. Then $F_x = 2x$, $F_y = 4y$, $F_z = -6z$, so $F_x(2, -1, 1) = 4$,
$F_y(2, -1, 1) = -4$, $F_z(2, -1, 1) = -6$. From Equation 11.6.19, an equation of the tangent plane is
$4(x - 2) - 4(y + 1) - 6(z - 1) = 0$ or equivalently $2x - 2y - 3z = 3$.

(b) From Equation 11.6.20, symmetric equations for the normal line are $\dfrac{x - 2}{4} = \dfrac{y + 1}{-4} = \dfrac{z - 1}{-6}$.

28. (a) Let $F(x, y, z) = xy + yz + zx$. Then $F_x = y + z$, $F_y = x + z$, $F_z = x + y$, so
$F_x(1, 1, 1) = F_y(1, 1, 1) = F_z(1, 1, 1) = 2$. From Equation 11.6.19, an equation of the tangent plane is
$2(x - 1) + 2(y - 1) + 2(z - 1) = 0$ or equivalently $x + y + z = 3$.

(b) From Equations 11.6.20, symmetric equations for the normal line are $\dfrac{x - 1}{2} = \dfrac{y - 1}{2} = \dfrac{z - 1}{2}$ or equivalently
$x - y = z$.

29. (a) $\mathbf{r}(u, v) = (u + v)\,\mathbf{i} + u^2\,\mathbf{j} + v^2\,\mathbf{k}$ and the point $(3, 4, 1)$ corresponds to $u = 2$, $v = 1$. Then $\mathbf{r}_u = \mathbf{i} + 2u\,\mathbf{j}$ $\Rightarrow$
$\mathbf{r}_u(2, 1) = \mathbf{i} + 4\,\mathbf{j}$ and $\mathbf{r}_v = \mathbf{i} + 2v\,\mathbf{k}$ $\Rightarrow$ $\mathbf{r}_v(2, 1) = \mathbf{i} + 2\,\mathbf{j}$. A normal vector to the surface at $(3, 4, 1)$ is
$\mathbf{r}_u \times \mathbf{r}_v = 8\,\mathbf{i} - 2\,\mathbf{j} - 4\,\mathbf{k}$, so an equation of the tangent plane there is $8(x - 3) - 2(y - 4) - 4(z - 1) = 0$ or
equivalently $4x - y - 2z = 6$.

(b) A direction vector for the normal line through $(3, 4, 1)$ is $8\,\mathbf{i} - 2\,\mathbf{j} - 4\,\mathbf{k}$, so a vector equation is
$\mathbf{r}(t) = (3\mathbf{i} + 4\mathbf{j} + \mathbf{k}) + t\,(8\,\mathbf{i} - 2\,\mathbf{j} - 4\,\mathbf{k})$, and the corresponding parametric equations are $x = 3 + 8t$,
$y = 4 - 2t$, $z = 1 - 4t$.

30. Let $f(x, y) = x^3 + 2xy$. Then $f_x(x, y) = 3x^2 + 2y$ and
$f_y(x, y) = 2x$, so $f_x(1, 2) = 7$, $f_y(1, 2) = 2$ and an
equation of the tangent plane is $z - 5 = 7(x - 1) + 2(y - 2)$
or $7x + 2y - z = 6$. The normal line is given by
$\dfrac{x - 1}{7} = \dfrac{y - 2}{2} = \dfrac{z - 5}{-1}$ or $x = 7t + 1$, $y = 2t + 2$,
$z = -t + 5$.
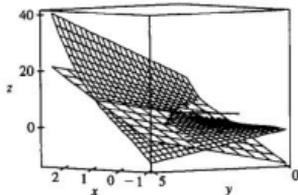
31. $F(x, y, z) = x^2 + y^2 + z^2$, $\nabla F(x_0, y_0, z_0) = \langle 2x_0, 2y_0, 2z_0 \rangle = k\langle 2, 1, -3 \rangle$ for $x_0 = k$, $y_0 = \frac{1}{2}k$ and $z_0 = -\frac{3}{2}k$.
But $x_0^2 + y_0^2 + z_0^2 = 1$, so $\frac{7}{2}k^2 = 1$ and $k = \pm\sqrt{\frac{2}{7}}$. Hence there are two such points: $\left(\pm\sqrt{\frac{2}{7}}, \pm\frac{1}{\sqrt{14}}, \mp\frac{3}{\sqrt{14}}\right)$.

32. $z = x^2\tan^{-1}y$ $\Rightarrow$ $dz = (2x\tan^{-1}y)\,dx + [x^2/(y^2 + 1)]\,dy$

33. $f(x, y, z) = x^3\sqrt{y^2 + z^2}$ $\Rightarrow$ $f_x(x, y, z) = 3x^2\sqrt{y^2 + z^2}$, $f_y(x, y, z) = \dfrac{yx^3}{\sqrt{y^2 + z^2}}$, and
$f_z(x, y, z) = \dfrac{zx^3}{\sqrt{y^2 + z^2}}$, so $f(2, 3, 4) = 8(5) = 40$, $f_x(2, 3, 4) = 3(4)\sqrt{25} = 60$, $f_y(2, 3, 4) = \dfrac{3(8)}{\sqrt{25}} = \dfrac{24}{5}$, and
$f_z(2, 3, 4) = \dfrac{4(8)}{\sqrt{25}} = \dfrac{32}{5}$. Then the linear approximation of f at $(2, 3, 4)$ is

$$f(x, y, z) \approx f(2, 3, 4) + f_x(2, 3, 4)(x - 2) + f_y(2, 3, 4)(y - 3) + f_z(2, 3, 4)(z - 4)$$
$$= 40 + 60(x - 2) + \tfrac{24}{5}(y - 3) + \tfrac{32}{5}(z - 4) = 60x + \tfrac{24}{5}y + \tfrac{32}{5}z - 120$$

Then

$$(1.98)^3\sqrt{(3.01)^2 + (3.97)^2} = f(1.98, 3.01, 3.97) \approx 60(1.98) + \tfrac{24}{5}(3.01) + \tfrac{32}{5}(3.97) - 120$$
$$= 38.656$$

34. (a) $dA = \dfrac{\partial A}{\partial x}\,dx + \dfrac{\partial A}{\partial y}\,dy = \tfrac{1}{2}y\,dx + \tfrac{1}{2}x\,dy$ and $|\Delta x| \le 0.002$, $|\Delta y| \le 0.002$. Thus the maximum error in the

calculated area is about $dA = 6(0.002) + \tfrac{5}{2}(0.002) = 0.017 \text{ m}^2$ or 170 cm^2.

(b) $z = \sqrt{x^2 + y^2}$, $dz = \dfrac{x}{\sqrt{x^2 + y^2}}\,dx + \dfrac{y}{\sqrt{x^2 + y^2}}\,dy$ and $|\Delta x| \le 0.002$, $|\Delta y| \le 0.002$. Thus the maximum

error in the calculated hypotenuse length is about $dz = \tfrac{5}{13}(0.002) + \tfrac{12}{13}(0.002) = \tfrac{0.17}{65} \approx 0.0026$ m or

0.26 cm.

35. $\dfrac{dw}{dt} = \dfrac{1}{2\sqrt{x}}\left(2e^{2t}\right) + \dfrac{2y}{z}\left(3t^2 + 4\right) + \dfrac{-y^2}{z^2}\left(2t\right) = e^t + \dfrac{2y}{z}\left(3t^2 + 4\right) - 2t\,\dfrac{y^2}{z^2}$

36. $\dfrac{\partial z}{\partial u} = (-y\sin xy - y\sin x)(2u) + (-x\sin xy + \cos x) = \cos x - 2uy\sin x - (\sin xy)(x + 2uy)$,
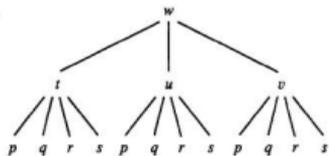
$\dfrac{\partial z}{\partial v} = (-y\sin xy - y\sin x)(1) + (-x\sin xy + \cos x)(-2v) = -2v\cos x + (\sin xy)(2vx - y) - y\sin x$

37. By the Chain Rule, $\dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial s}$. When $s = 1$ and $t = 2$, $x = g(1, 2) = 3$ and $y = h(1, 2) = 6$, so

$\dfrac{\partial z}{\partial s} = f_x(3, 6)g_s(1, 2) + f_y(3, 6)\,h_s(1, 2) = (7)(-1) + (8)(-5) = -47$. Similarly, $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial t}$, so

$\dfrac{\partial z}{\partial t} = f_x(3, 6)g_t(1, 2) + f_y(3, 6)\,h_t(1, 2) = (7)(4) + (8)(10) = 108$.

38.

Using the tree diagram as a guide, we have

$\dfrac{\partial w}{\partial p} = \dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial p} + \dfrac{\partial w}{\partial u}\dfrac{\partial u}{\partial p} + \dfrac{\partial w}{\partial v}\dfrac{\partial v}{\partial p}$

$\dfrac{\partial w}{\partial q} = \dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial q} + \dfrac{\partial w}{\partial u}\dfrac{\partial u}{\partial q} + \dfrac{\partial w}{\partial v}\dfrac{\partial v}{\partial q}$

$\dfrac{\partial w}{\partial r} = \dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial r} + \dfrac{\partial w}{\partial u}\dfrac{\partial u}{\partial r} + \dfrac{\partial w}{\partial v}\dfrac{\partial v}{\partial r}$

$\dfrac{\partial w}{\partial s} = \dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial s} + \dfrac{\partial w}{\partial u}\dfrac{\partial u}{\partial s} + \dfrac{\partial w}{\partial v}\dfrac{\partial v}{\partial s}$

39. $\dfrac{\partial z}{\partial x} = 2xf'(x^2 - y^2)$, $\dfrac{\partial z}{\partial y} = 1 - 2yf'(x^2 - y^2)$ $\quad\left[\text{where } f' = \dfrac{df}{d(x^2 - y^2)}\right]$. Then

$y\,\dfrac{\partial z}{\partial x} + x\,\dfrac{\partial z}{\partial y} = 2xyf'(x^2 - y^2) + x - 2xyf'(x^2 - y^2) = x$.

40. $A = \tfrac{1}{2}xy\sin\theta$, $dx/dt = 3$, $dy/dt = -2$, $d\theta/dt = 0.05$, and

$\dfrac{dA}{dt} = \dfrac{1}{2}\left[(y\sin\theta)\dfrac{dx}{dt} + (x\sin\theta)\dfrac{dy}{dt} + (xy\cos\theta)\dfrac{d\theta}{dt}\right]$. So when $x = 40$, $y = 50$ and $\theta = \tfrac{\pi}{6}$,

$\dfrac{dA}{dt} = \dfrac{1}{2}\left[(25)(3) + (20)(-2) + (1000\sqrt{3})(0.05)\right] = \dfrac{35 + 50\sqrt{3}}{2} \approx 60.8 \text{ in}^2/\text{s}$.