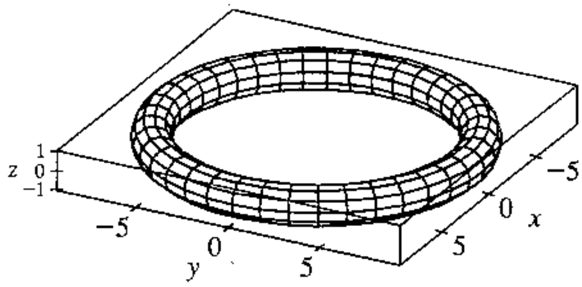
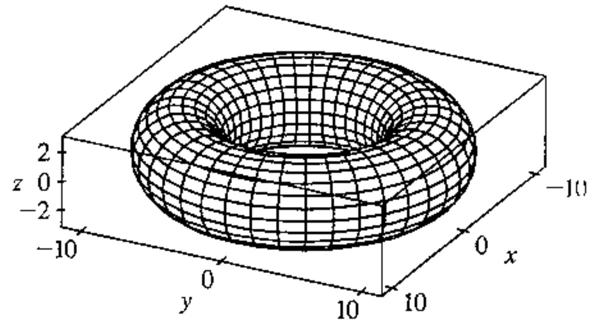
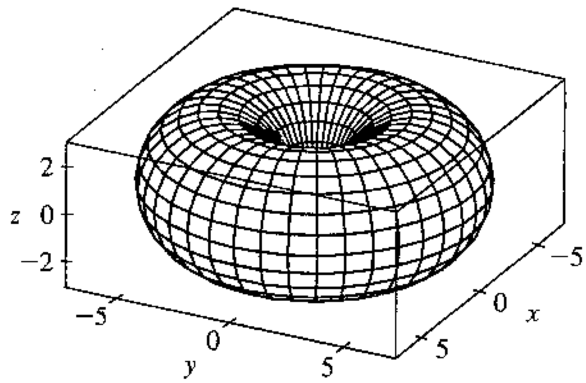
(b)



$a = 1, b = 8$



$a = 3, b = 8$



$a = 3, b = 4$

10 Review

• CONCEPT CHECK •

1. A vector function is a function whose domain is a set of real numbers and whose range is a set of vectors. To find the derivative or integral, we can differentiate or integrate each component of the vector function.

2. The tip of the moving vector $\mathbf{r}(t)$ of a continuous vector function traces out a space curve.

3. (a) A curve represented by the vector function $\mathbf{r}(t)$ is smooth if $\mathbf{r}'(t)$ is continuous and $\mathbf{r}'(t) \neq \mathbf{0}$ on its parametric domain (except possibly at the endpoints).

(b) The tangent vector to a smooth curve at a point P with position vector $\mathbf{r}(t)$ is the vector $\mathbf{r}'(t)$. The tangent line at P is the line through P parallel to the tangent vector $\mathbf{r}'(t)$. The unit tangent vector is $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$.

4. (a) – (f) See Theorem 10.2.3.

5. Use Formula 10.3.2, or equivalently 10.3.3.

6. (a) The curvature of a curve is $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right|$ where $\mathbf{T}$ is the unit tangent vector.

(b) $\kappa(t) = \left|\dfrac{\mathbf{T}'(t)}{\mathbf{r}'(t)}\right|$ (c) $\kappa(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$ (d) $\kappa(x) = \dfrac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}$

7. (a) The unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$. The binormal vector: $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$.

(b) See the discussion preceding Example 7 in Section 10.3.

8. (a) If $\mathbf{r}(t)$ is the position vector of the particle on the space curve, the velocity $\mathbf{v}(t) = \mathbf{r}'(t)$, the speed is given by $|\mathbf{v}(t)|$, and the acceleration $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$.

(b) $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ where $a_T = v'$ and $a_N = \kappa v^2$.

9. See the statement of Kepler's Laws on page 731.

10. See the discussion on pages 736 and 737.

▲ TRUE–FALSE QUIZ ▲

1. True. If we reparametrize the curve by replacing $u = t^3$, we have $\mathbf{r}(u) = u\,\mathbf{i} + 2u\,\mathbf{j} + 3u\,\mathbf{k}$, which is a line through the origin with direction vector $\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$.

2. True. $\mathbf{r}'(t) = \langle 1, 3t^2, 5t^4 \rangle$ is continuous for all t (since its component functions are each continuous) and since $x'(t) = 1$, we have $\mathbf{r}'(t) \neq \mathbf{0}$ for all t.

3. False. $\mathbf{r}'(t) = \langle -\sin t, 2t, 4t^3 \rangle$, and since $\mathbf{r}'(0) = \langle 0, 0, 0 \rangle = \mathbf{0}$, the curve is not smooth.

4. True. See Theorem 10.2.2.

5. False. By Formula 5 of Theorem 10.2.3, $\dfrac{d}{dt}\left[\mathbf{u}(t) \times \mathbf{v}(t)\right] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$.

6. False. For example, let $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$. Then $|\mathbf{r}(t)| = \sqrt{\cos^2 t + \sin^2 t} = 1 \quad\Rightarrow\quad \dfrac{d}{dt}\,|\mathbf{r}(t)| = 0$, but $|\mathbf{r}'(t)| = |\langle -\sin t, \cos t \rangle| = \sqrt{(-\sin t)^2 + \cos^2 t} = 1$.

7. False. κ is the magnitude of the rate of change of the unit tangent vector $\mathbf{T}$ with respect to arc length s, not with respect to t.

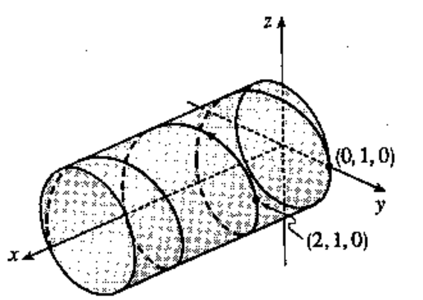
8. False. The binormal vector, by the definition given in Section 10.3, is $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t) = -\left[\mathbf{N}(t) \times \mathbf{T}(t)\right]$.

9. True. See the discussion preceding Example 7 in Section 10.3.

10. False. For example, $\mathbf{r}_1(t) = \langle t, t \rangle$ and $\mathbf{r}_2(t) = \langle 2t, 2t \rangle$ both represent the same plane curve (the line $y = x$), but the tangent vector $\mathbf{r}_1'(t) = \langle 1, 1 \rangle$ for all t, while $\mathbf{r}_2'(t) = \langle 2, 2 \rangle$. In fact, different parametrizations give parallel tangent vectors at a point, but their magnitudes may differ.

◆ EXERCISES ◆

1. (a) The corresponding parametric equations for the curve are $x = t$, $y = \cos \pi t$, $z = \sin \pi t$. Since $y^2 + z^2 = 1$, the curve is contained in a circular cylinder with axis the x-axis. Since $x = t$, the curve is a helix.



(b) $\mathbf{r}(t) = t\,\mathbf{i} + \cos \pi t\,\mathbf{j} + \sin \pi t\,\mathbf{k} \quad\Rightarrow\quad \mathbf{r}'(t) = \mathbf{i} - \pi \sin \pi t\,\mathbf{j} + \pi \cos \pi t\,\mathbf{k} \quad\Rightarrow$
$\mathbf{r}''(t) = -\pi^2 \cos \pi t\,\mathbf{j} - \pi^2 \sin \pi t\,\mathbf{k}$

2. (a) The expressions $\sqrt{2-t}$, $(e^t-1)/t$, and $\ln(t+1)$ are all defined when $2-t \geq 0 \quad \Rightarrow \quad t \leq 2$, $t \neq 0$, and $t+1>0 \quad \Rightarrow \quad t>-1$. Thus the domain of $\mathbf{r}$ is $(-1,0) \cup (0,2]$.

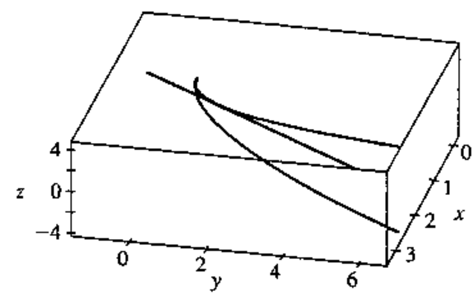
(b) $\lim\limits_{t\to 0} \mathbf{r}(t) = \left\langle \lim\limits_{t\to 0} \sqrt{2-t}, \lim\limits_{t\to 0} \dfrac{e^t-1}{t}, \lim\limits_{t\to 0} \ln(t+1) \right\rangle = \left\langle \sqrt{2-0}, \lim\limits_{t\to 0} \dfrac{e^t}{1}, \ln(0+1) \right\rangle = \langle \sqrt{2}, 1, 0 \rangle$

(using l'Hospital's Rule in the y-component).

(c) $\mathbf{r}'(t) = \left\langle \dfrac{d}{dt}\sqrt{2-t}, \dfrac{d}{dt}\dfrac{e^t-1}{t}, \dfrac{d}{dt}\ln(t+1) \right\rangle = \left\langle -\dfrac{1}{2\sqrt{2-t}}, \dfrac{te^t-e^t+1}{t^2}, \dfrac{1}{t+1} \right\rangle$

3. The projection of the curve C of intersection onto the xy-plane is the circle $x^2+y^2=16$, $z=0$. So we can write $x=4\cos t$, $y=4\sin t$, $0 \leq t \leq 2\pi$. From the equation of the plane, we have $z=5-x=5-4\cos t$, so parametric equations for C are $x=4\cos t$, $y=4\sin t$, $z=5-4\cos t$, $0 \leq t \leq 2\pi$, and the corresponding vector function is $\mathbf{r}(t) = 4\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + (5-4\cos t)\,\mathbf{k}$, $0 \leq t \leq 2\pi$.

4. The curve is given by $\mathbf{r}(t) = \langle t^2, t^4, t^3 \rangle$, so $\mathbf{r}'(t) = \langle 2t, 4t^3, 3t^2 \rangle$. The point $(1,1,1)$ corresponds to $t=1$, so the tangent vector there is $\mathbf{r}'(1) = \langle 2,4,3 \rangle$. Then the tangent line has direction vector $\langle 2,4,3 \rangle$ and includes the point $(1,1,1)$, so parametric equations are $x=1+2t$, $y=1+4t$, $z=1+3t$.



5. $\displaystyle\int_0^1 \left(t^2\,\mathbf{i} + t\cos \pi t\,\mathbf{j} + \sin \pi t\,\mathbf{k}\right) dt = \left(\int_0^1 t^2\,dt\right)\mathbf{i} + \left(\int_0^1 t\cos \pi t\,dt\right)\mathbf{j} + \left(\int_0^1 \sin \pi t\,dt\right)\mathbf{k}$

$$= \left[\tfrac{1}{3}t^3\right]_0^1 \mathbf{i} + \left(\left.\tfrac{t}{\pi}\sin \pi t\right]_0^1 - \int_0^1 \tfrac{1}{\pi}\sin \pi t\,dt\right)\mathbf{j} + \left[-\tfrac{1}{\pi}\cos \pi t\right]_0^1 \mathbf{k}$$

$$= \tfrac{1}{3}\mathbf{i} + \left[\tfrac{1}{\pi^2}\cos \pi t\right]_0^1 \mathbf{j} + \tfrac{2}{\pi}\mathbf{k} = \tfrac{1}{3}\mathbf{i} - \tfrac{2}{\pi^2}\mathbf{j} + \tfrac{2}{\pi}\mathbf{k}$$

where we integrated by parts in the y-component.

6. (a) C intersects the xz-plane where $y=0 \quad \Rightarrow \quad 2t-1=0 \quad \Rightarrow \quad t=\frac{1}{2}$, so the point is $\left(2-\left(\frac{1}{2}\right)^3, 0, \ln\frac{1}{2}\right) = \left(\frac{15}{8}, 0, -\ln 2\right)$.

(b) The curve is given by $\mathbf{r}(t) = \langle 2-t^3, 2t-1, \ln t \rangle$, so $\mathbf{r}'(t) = \langle -3t^2, 2, 1/t \rangle$. The point $(1,1,0)$ corresponds to $t=1$, so the tangent vector there is $\mathbf{r}'(1) = \langle -3,2,1 \rangle$. Then the tangent line has direction vector $\langle -3,2,1 \rangle$ and includes the point $(1,1,0)$, so parametric equations are $x=1-3t$, $y=1+2t$, $z=t$.

(c) The normal plane has normal vector $\mathbf{r}'(1) = \langle -3,2,1 \rangle$ and equation $-3(x-1)+2(y-1)+z=0$ or $3x-2y-z=1$.

7. $t=1$ at $(1,4,2)$ and $t=4$ at $(2,1,17)$, so

$$L = \int_1^4 \sqrt{\frac{1}{4t} + \frac{16}{t^4} + 4t^2}\,dt$$

$$\approx \frac{4-1}{3\cdot 4}\left[\sqrt{\tfrac{1}{4}+16+4} + 4\cdot\sqrt{\frac{1}{4\cdot\frac{7}{4}} + \frac{16}{\left(\frac{7}{4}\right)^4} + 4\left(\tfrac{7}{4}\right)^2} + 2\cdot\sqrt{\frac{1}{4\cdot\frac{10}{4}} + \frac{16}{\left(\frac{10}{4}\right)^4} + 4\left(\tfrac{10}{4}\right)^2}\right.$$

$$\left. + 4\cdot\sqrt{\frac{1}{4\cdot\frac{13}{4}} + \frac{16}{\left(\frac{13}{4}\right)^4} + 4\left(\tfrac{13}{4}\right)^2} + \sqrt{\frac{1}{4\cdot 4} + \frac{16}{4^4} + 4\cdot 4^2}\right]$$

≈ 15.9241

8. $\mathbf{r}'(t) = \left\langle 3t^{1/2}, -2\sin 2t, 2\cos 2t \right\rangle$, $|\mathbf{r}'(t)| = \sqrt{9t + 4(\sin^2 2t + \cos^2 2t)} = \sqrt{9t+4}$. Thus
$L = \int_0^1 \sqrt{9t+4}\,dt = \int_4^{13} \frac{1}{9} u^{1/2}\,du = \frac{1}{9} \cdot \frac{2}{3} u^{3/2} \Big]_4^{13} = \frac{2}{27}\left(13^{3/2} - 8\right)$.

9. The angle of intersection of the two curves, θ, is the angle between their respective tangents at the point of intersection. For both curves the point $(1,0,0)$ occurs when $t = 0$. $\mathbf{r}_1'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \mathbf{k} \;\Rightarrow$ $\mathbf{r}_1'(0) = \mathbf{j} + \mathbf{k}$ and $\mathbf{r}_2'(t) = \mathbf{i} + 2t\,\mathbf{j} + 3t^2\,\mathbf{k} \;\Rightarrow\; \mathbf{r}_2'(0) = \mathbf{i}$. $\mathbf{r}_1'(0) \cdot \mathbf{r}_2'(0) = (\mathbf{j}+\mathbf{k}) \cdot \mathbf{i} = 0$. Therefore, the curves intersect in a right angle, that is, $\theta = \frac{\pi}{2}$.

10. The parametric value corresponding to the point $(1,0,1)$ is $t = 0$.
$\mathbf{r}'(t) = e^t\,\mathbf{i} + e^t(\cos t + \sin t)\mathbf{j} + e^t(\cos t - \sin t)\mathbf{k}$
$\Rightarrow \quad |\mathbf{r}'(t)| = e^t\sqrt{1 + (\cos t + \sin t)^2 + (\cos t - \sin t)^2} = \sqrt{3}\,e^t$
and $s(t) = \int_0^t e^u \sqrt{3}\,du = \sqrt{3}(e^t - 1) \quad\Rightarrow\quad t = \ln\left(1 + \frac{1}{\sqrt{3}}s\right)$. Therefore,
$\mathbf{r}(t(s)) = \left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{i} + \left(1 + \frac{1}{\sqrt{3}}s\right)\sin\ln\left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{j} + \left(1 + \frac{1}{\sqrt{3}}s\right)\cos\ln\left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{k}$.

11. (a) $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \dfrac{\langle t^2, t, 1\rangle}{|\langle t^2, t, 1\rangle|} = \dfrac{\langle t^2, t, 1\rangle}{\sqrt{t^4 + t^2 + 1}}$

(b)
$$\begin{aligned}\mathbf{T}'(t) &= -\tfrac{1}{2}\left(t^4 + t^2 + 1\right)^{-3/2}\left(4t^3 + 2t\right)\left\langle t^2, t, 1\right\rangle + \left(t^4 + t^2 + 1\right)^{-1/2}\langle 2t, 1, 0\rangle \\ &= \frac{-2t^3 - t}{\left(t^4 + t^2 + 1\right)^{3/2}}\left\langle t^2, t, 1\right\rangle + \frac{1}{\left(t^4 + t^2 + 1\right)^{1/2}}\langle 2t, 1, 0\rangle \\ &= \frac{\left\langle -2t^5 - t^3, -2t^4 - t^2, -2t^3 - t\right\rangle + \left\langle 2t^5 + 2t^3 + 2t, t^4 + t^2 + 1, 0\right\rangle}{\left(t^4 + t^2 + 1\right)^{3/2}} \\ &= \frac{\left\langle 2t, -t^4 + 1, -2t^3 - t\right\rangle}{\left(t^4 + t^2 + 1\right)^{3/2}}\end{aligned}$$

$$|\mathbf{T}'(t)| = \frac{\sqrt{4t^2 + t^8 - 2t^4 + 1 + 4t^6 + 4t^4 + t^2}}{\left(t^4 + t^2 + 1\right)^{3/2}} = \frac{\sqrt{t^8 + 4t^6 + 2t^4 + 5t^2}}{\left(t^4 + t^2 + 1\right)^{3/2}}, \text{ and}$$

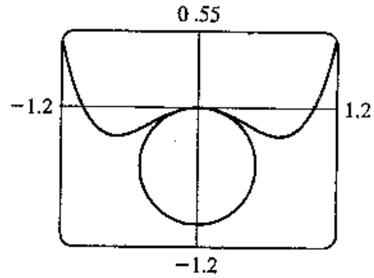
$$\mathbf{N}(t) = \frac{\left\langle 2t, 1 - t^4, -2t^3 - t\right\rangle}{\sqrt{t^8 + 4t^6 + 2t^4 + 5t^2}}.$$

(c) $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{\sqrt{t^8 + 4t^6 + 2t^4 + 5t^2}}{\left(t^4 + t^2 + 1\right)^2}$

12. Using Exercise 10.3.32, we have $\mathbf{r}'(t) = \langle -3\sin t, 4\cos t\rangle$, $\mathbf{r}''(t) = \langle -3\cos t, -4\sin t\rangle$,
$|\mathbf{r}'(t)|^3 = \left(\sqrt{9\sin^2 t + 4\cos^2 t}\right)^3$ and then
$$\kappa(t) = \frac{|(-3\sin t)(-4\sin t) - (4\cos t)(-3\cos t)|}{\left(9\sin^2 t + 16\cos^2 t\right)^{3/2}} = \frac{12}{\left(9\sin^2 t + 16\cos^2 t\right)^{3/2}}.$$
At $(3,0)$, $t = 0$ and $\kappa(0) = 12/(16)^{3/2} = \frac{12}{64} = \frac{3}{16}$. At $(0,4)$, $t = \frac{\pi}{2}$ and $\kappa\left(\frac{\pi}{2}\right) = 12/9^{3/2} = \frac{12}{27} = \frac{4}{9}$.

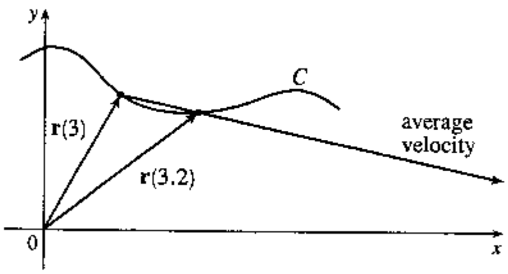
13. $y' = 4x^3$, $y'' = 12x^2$ and $\kappa(x) = \dfrac{|y''|}{\left[1 + (y')^2\right]^{3/2}} = \dfrac{|12x^2|}{\left(1 + 16x^6\right)^{3/2}}$, so $\kappa(1) = \dfrac{12}{17^{3/2}}$.

14. $\kappa(x) = \dfrac{|12x^2 - 2|}{\left[1 + (4x^3 - 2x)^2\right]^{3/2}} \Rightarrow \kappa(0) = 2$. So the osculating circle has radius $\frac{1}{2}$ and center $\left(0, -\frac{1}{2}\right)$. Thus its equation is $x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$.



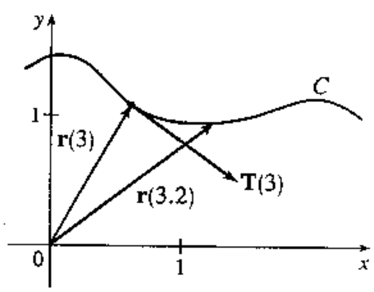
15. $\mathbf{r}(t) = \langle \sin 2t, t, \cos 2t \rangle \Rightarrow \mathbf{r}'(t) = \langle 2\cos 2t, 1, -2\sin 2t \rangle \Rightarrow \mathbf{T}(t) = \frac{1}{\sqrt{5}} \langle 2\cos 2t, 1, -2\sin 2t \rangle \Rightarrow$ $\mathbf{T}'(t) = \frac{1}{\sqrt{5}} \langle -4\sin 2t, 0, -4\cos 2t \rangle \Rightarrow \mathbf{N}(t) = \langle -\sin 2t, 0, -\cos 2t \rangle$. So $\mathbf{N} = \mathbf{N}(\pi) = \langle 0, 0, -1 \rangle$ and $\mathbf{B} = \mathbf{T} \times \mathbf{N} = \frac{1}{\sqrt{5}} \langle -1, 2, 0 \rangle$. So a normal to the osculating plane is $\langle -1, 2, 0 \rangle$ and an equation is $-1(x - 0) + 2(y - \pi) + 0(z - 1) = 0$ or $x - 2y + 2\pi = 0$.

16. (a) The average velocity over $[3, 3.2]$ is given by $\dfrac{\mathbf{r}(3.2) - \mathbf{r}(3)}{3.2 - 3} = 5[\mathbf{r}(3.2) - \mathbf{r}(3)]$, so we draw a vector with the same direction but 5 times the length of the vector $[\mathbf{r}(3.2) - \mathbf{r}(3)]$.



(b) $\mathbf{v}(3) = \mathbf{r}'(3) = \lim\limits_{h \to 0} \dfrac{\mathbf{r}(3 + h) - \mathbf{r}(3)}{h}$.

(c) $\mathbf{T}(3) = \dfrac{\mathbf{r}'(3)}{|\mathbf{r}'(3)|}$, a unit vector in the same direction as $\mathbf{r}'(3)$, that is, parallel to the tangent line to the curve at $\mathbf{r}(3)$, pointing in the direction corresponding to increasing t, and with length 1.



17. $\mathbf{r}(t) = t\ln t\,\mathbf{i} + t\,\mathbf{j} + e^{-t}\,\mathbf{k}$, $\mathbf{v}(t) = \mathbf{r}'(t) = (1 + \ln t)\,\mathbf{i} + \mathbf{j} - e^{-t}\,\mathbf{k}$,
$|\mathbf{v}(t)| = \sqrt{(1 + \ln t)^2 + 1^2 + (-e^{-t})^2} = \sqrt{2 + 2\ln t + (\ln t)^2 + e^{-2t}}$, $\mathbf{a}(t) = \mathbf{v}'(t) = \frac{1}{t}\,\mathbf{i} + e^{-t}\,\mathbf{k}$.

18. $\mathbf{v}(t) = \int \mathbf{a}(t)\,dt = \int \left(6t\,\mathbf{i} + 12t^2\,\mathbf{j} - 6t\,\mathbf{k}\right) dt = 3t^2\,\mathbf{i} + 4t^3\,\mathbf{j} - 3t^2\,\mathbf{k} + \mathbf{C}$, but
$\mathbf{i} - \mathbf{j} + 3\mathbf{k} = \mathbf{v}(0) = \mathbf{0} + \mathbf{C}$, so $\mathbf{C} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{v}(t) = \left(3t^2 + 1\right)\mathbf{i} + \left(4t^3 - 1\right)\mathbf{j} + \left(3 - 3t^2\right)\mathbf{k}$.
$\mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \left(t^3 + t\right)\mathbf{i} + \left(t^4 - t\right)\mathbf{j} + \left(3t - t^3\right)\mathbf{k} + \mathbf{D}$. But $\mathbf{r}(0) = \mathbf{0}$, so $\mathbf{D} = \mathbf{0}$ and
$\mathbf{r}(t) = \left(t^3 + t\right)\mathbf{i} + \left(t^4 - t\right)\mathbf{j} + \left(3t - t^3\right)\mathbf{k}$.

19. We set up the axes so that the shot leaves the athlete's hand 7 ft above the origin. Then we are given $\mathbf{r}(0) = 7\mathbf{j}$, $|\mathbf{v}(0)| = 43$ ft/s, and $\mathbf{v}(0)$ has direction given by a $45\,^\circ$ angle of elevation. Then a unit vector in the direction of $\mathbf{v}(0)$ is $\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j}) \Rightarrow \mathbf{v}(0) = \frac{43}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$. Assuming air resistance is negligible, the only external force is due to gravity, so as in Example 10.4.5 we have $\mathbf{a} = -g\,\mathbf{j}$ where here $g \approx 32$ ft/s^2. Since $\mathbf{v}'(t) = \mathbf{a}(t)$, we integrate, giving $\mathbf{v}(t) = -gt\,\mathbf{j} + \mathbf{C}$ where $\mathbf{C} = \mathbf{v}(0) = \frac{43}{\sqrt{2}}(\mathbf{i}+\mathbf{j}) \Rightarrow \mathbf{v}(t) = \frac{43}{\sqrt{2}}\,\mathbf{i} + \left(\frac{43}{\sqrt{2}} - gt\right)\mathbf{j}$. Since $\mathbf{r}'(t) = \mathbf{v}(t)$ we integrate again, so $\mathbf{r}(t) = \frac{43}{\sqrt{2}}t\,\mathbf{i} + \left(\frac{43}{\sqrt{2}}t - \frac{1}{2}gt^2\right)\mathbf{j} + \mathbf{D}$. But

$\mathbf{D} = \mathbf{r}(0) = 7\,\mathbf{j} \quad \Rightarrow \quad \mathbf{r}(t) = \frac{43}{\sqrt{2}}t\,\mathbf{i} + \left(\frac{43}{\sqrt{2}}t - \frac{1}{2}gt^2 + 7\right)\mathbf{j}$.

(a) At 2 seconds, the shot is at $\mathbf{r}(2) = \frac{43}{\sqrt{2}}(2)\mathbf{i} + \left(\frac{43}{\sqrt{2}}(2) - \frac{1}{2}g(2)^2 + 7\right)\mathbf{j} \approx 60.8\,\mathbf{i} + 3.8\,\mathbf{j}$, so the shot is about 3.8 ft above the ground, at a horizontal distance of 60.8 ft from the athlete.

(b) The shot reaches its maximum height when the vertical component of velocity is 0: $\frac{43}{\sqrt{2}} - gt = 0 \Rightarrow$ $t = \dfrac{43}{\sqrt{2}g} \approx 0.95$ s. Then $\mathbf{r}(0.95) \approx 28.9\,\mathbf{i} + 21.4\,\mathbf{j}$, so the maximum height is approximately 21.4 ft.

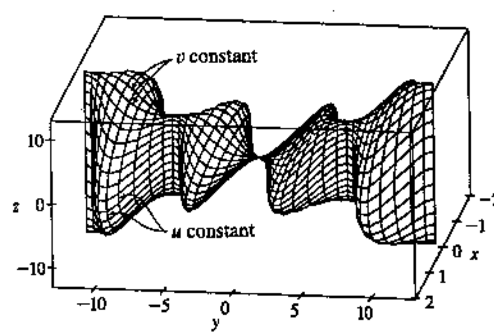
(c) The shot hits the ground when the vertical component of $\mathbf{r}(t)$ is 0, so $\frac{43}{\sqrt{2}}t - \frac{1}{2}gt^2 + 7 = 0 \Rightarrow$ $-16t^2 + \frac{43}{\sqrt{2}}t + 7 = 0 \Rightarrow t \approx 2.11$ s. $\mathbf{r}(2.11) \approx 64.2\,\mathbf{i} - 0.08\,\mathbf{j}$, thus the shot lands approximately 64.2 ft from the athlete.

20. $\mathbf{r}'(t) = \mathbf{i} + 2\,\mathbf{j} + 2t\,\mathbf{k}$, $\mathbf{r}''(t) = 2\,\mathbf{k}$, $|\mathbf{r}'(t)| = \sqrt{1+4+4t^2} = \sqrt{4t^2+5}$.

Then $a_T = \dfrac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|} = \dfrac{4t}{\sqrt{4t^2+5}}$ and $a_N = \dfrac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|} = \dfrac{|4\mathbf{i}-2\mathbf{j}|}{\sqrt{4t^2+5}} = \dfrac{2\sqrt{5}}{\sqrt{4t^2+5}}$.

21. From Example 4 in Section 10.5, a parametric representation of the sphere $x^2+y^2+z^2 = 4$ is $x = 2\sin\phi\cos\theta$, $y = 2\sin\phi\sin\theta$, $z = 2\cos\phi$ with $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$. We can restrict the surface to that portion between the planes $z = 1$ and $z = -1$ by restricting $-1 \le z \le 1 \Rightarrow -1 \le 2\cos\phi \le 1 \Rightarrow \frac{\pi}{3} \le \phi \le \frac{2\pi}{3}$.

22. $x = (1-\cos u)\sin v$, $y = u$, $z = (u - \sin u)\cos v$. We plot the portion of the surface corresponding to the parametric domain $-4\pi \le u \le 4\pi$, $0 \le v \le 2\pi$. If u is held constant, x and z give the equation of an ellipse in the plane $y = u$, thus the grid curves are the vertically oriented ellipses we see. Note that when $u = 2k\pi$, k an integer, we have $x = 0$ and z ranges from $-2k\pi$ to $2k\pi$, representing a line segment parallel to the z-axis in the plane $y = 2k\pi$. These are the vertical "seams" we see on the surface. If v is held constant, y is free to vary, so the grid curves running lengthwise along the surface correspond to keeping v constant.



23. (a) Instead of proceeding directly, we use Formula 3 of Theorem 10.2.3: $\mathbf{r}(t) = t\,\mathbf{R}(t) \Rightarrow$
$\mathbf{v} = \mathbf{r}'(t) = \mathbf{R}(t) + t\,\mathbf{R}'(t) = \cos\omega t\,\mathbf{i} + \sin\omega t\,\mathbf{j} + t\,\mathbf{v}_d$.

(b) Using the same method as in part (a) and starting with $\mathbf{v} = \mathbf{R}(t) + t\,\mathbf{R}'(t)$, we have
$\mathbf{a} = \mathbf{v}' = \mathbf{R}'(t) + \mathbf{R}'(t) + t\,\mathbf{R}''(t) = 2\,\mathbf{R}'(t) + t\,\mathbf{R}''(t) = 2\,\mathbf{v}_d + t\,\mathbf{a}_d$.

(c) Here we have $\mathbf{r}(t) = e^{-t}\cos\omega t\,\mathbf{i} + e^{-t}\sin\omega t\,\mathbf{j} = e^{-t}\,\mathbf{R}(t)$. So, as in parts (a) and (b),

$$\mathbf{v} = \mathbf{r}'(t) = e^{-t}\,\mathbf{R}'(t) - e^{-t}\,\mathbf{R}(t) = e^{-t}\,[\mathbf{R}'(t) - \mathbf{R}(t)] \Rightarrow$$

$$\begin{aligned}\mathbf{a} = \mathbf{v}' &= e^{-t}\,[\mathbf{R}''(t) - \mathbf{R}'(t)] - e^{-t}\,[\mathbf{R}'(t) - \mathbf{R}(t)] = e^{-t}\,[\mathbf{R}''(t) - 2\,\mathbf{R}'(t) + \mathbf{R}(t)] \\ &= e^{-t}\,\mathbf{a}_d - 2e^{-t}\,\mathbf{v}_d + e^{-t}\,\mathbf{R}\end{aligned}$$

Thus, the Coriolis acceleration (the sum of the "extra" terms not involving $\mathbf{a}_d$) is $-2e^{-t}\,\mathbf{v}_d + e^{-t}\,\mathbf{R}$.

24. By the Fundamental Theorem of Calculus, $\mathbf{r}'(t) = \langle \sin(\pi t^2/2), \cos(\pi t^2/2)\rangle$, $|\mathbf{r}'(t)| = 1$ and so $\mathbf{T}(t) = \mathbf{r}'(t)$.
Thus $\mathbf{T}'(t) = \pi t\,\langle \sin(\pi t^2/2), \cos(\pi t^2/2)\rangle$ and the curvature is $\kappa = |\mathbf{T}'(t)| = \sqrt{(\pi t)^2(1)} = \pi\,|t|$.

25. (a) $$F(x) = \begin{cases} 1 & \text{if } x \le 0 \\ \sqrt{1-x^2} & \text{if } 0 < x < \frac{1}{\sqrt{2}} \\ -x + \sqrt{2} & \text{if } x \ge \frac{1}{\sqrt{2}} \end{cases} \Rightarrow F'(x) = \begin{cases} 0 & \text{if } x < 0 \\ -x/\sqrt{1-x^2} & \text{if } 0 < x < \frac{1}{\sqrt{2}} \\ -1 & \text{if } x > \frac{1}{\sqrt{2}} \end{cases} \Rightarrow$$

$$F''(x) = \begin{cases} 0 & \text{if } x < 0 \\ -1/(1-x^2)^{3/2} & \text{if } 0 < x < \frac{1}{\sqrt{2}} \\ 0 & \text{if } x > \frac{1}{\sqrt{2}} \end{cases}$$

since $\dfrac{d}{dx}\left[-x\left(1-x^2\right)^{-1/2}\right] = -\left(1-x^2\right)^{-1/2} - x^2\left(1-x^2\right)^{-3/2} = -\left(1-x^2\right)^{-3/2}$.

Now $\lim\limits_{x\to 0^+}\sqrt{1-x^2} = 1 = F(0)$ and $\lim\limits_{x\to(1/\sqrt{2})^-}\sqrt{1-x^2} = \frac{1}{\sqrt{2}} = F\left(\frac{1}{\sqrt{2}}\right)$, so F is continuous. Also, since $\lim\limits_{x\to 0^+} F'(x) = 0 = \lim\limits_{x\to 0^-} F'(x)$ and $\lim\limits_{x\to(1/\sqrt{2})^-} F'(x) = -1 = \lim\limits_{x\to(1/\sqrt{2})^+} F'(x)$, F' is continuous. But $\lim\limits_{x\to 0^+} F''(x) = -1 \ne 0 = \lim\limits_{x\to 0^-} F''(x)$, so F'' is not continuous at $x = 0$. (The same is true at $x = \frac{1}{\sqrt{2}}$.) So F does not have continuous curvature.

(b) Set $P(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$. The continuity conditions on P are $P(0) = 0$, $P(1) = 1$, $P'(0) = 0$ and $P'(1) = 1$. Also the curvature must be continuous. For $x \le 0$ and $x \ge 1$, $\kappa(x) = 0$; elsewhere $\kappa(x) = \dfrac{|P''(x)|}{\left(1 + [P'(x)]^2\right)^{3/2}}$, so we need $P''(0) = 0$ and $P''(1) = 0$.

The conditions $P(0) = P'(0) = P''(0) = 0$ imply that $d = e = f = 0$. The other conditions imply that $a + b + c = 1$, $5a + 4b + 3c = 1$, and $10a + 6b + 3c = 0$. From these, we find that $a = 3$, $b = -8$, and $c = 6$. Therefore $P(x) = 3x^5 - 8x^4 + 6x^3$. Since there was no solution with $a = 0$, this could not have been done with a polynomial of degree 4.