Math 21a Hourly 1

(Fall 2001)

1) ___ 2) ___ 3) ___ 4) ___ 5) ___ 6) ___ : Total ______

Name: ________________________________

Circle the name of your Section TF:

Arinkin(10)•Arinkin(12)•Bamberg•Cornut•Kaplan•Karu•Knill•Libine•Liu•Taubes•Williams

Instructions:

- Print your name in the line above and circle the name of your section TF.
- Answer each of the questions below on the same page as the question. If more space is needed, use the back of the facing page. Extra blank pages are also provided at the back of this packet.
- Do not detach pages from this exam packet or unstaple this packet.
- Please write neatly. Answers which are deemed illegible by the grader will not receive credit.
- No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students. Use only your brain and a pencil.
- You have 2 hours to complete your work.
- All numbered problems have identical maximal possible point count even as some are harder than others.
- Bold fact type is used below to indicate vectors.

By agreeing to take this exam, you are implicitly agreeing to procede fairly and honestly.

1. Put T next to each of the true statements below and put F next to the false ones. There is no need to justify your answer.
   a) There exist two non-zero vectors $\mathbf{v}$ and $\mathbf{w}$ with $\mathbf{v} \times \mathbf{w} = 0$ and $\mathbf{v} \bullet \mathbf{w} = 0$.
   b) The planes where $x + 3y + z = 1$ and $5x - 3y + 4z = 1$ intersect at right angles.
   c) All points (x, y, z) with $x^2 + y^2 + z^2 = 2x$ have distance 1 from $(1, 0, 0)$.
   d) The line parameterized as $t \to (1 - 4t, 1 - 6t, 1 - 8t)$ intersects the plane where $2x + 3y + 4z = 9$ in a right angle.
   e) The path parameterized as $t \to (1 + 2t^3, 1 + 5t^3, t^3)$ is a straight line.
   f) There are non-zero vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^3$ with $|\mathbf{v}| = 3|\mathbf{w}|$ and $\mathbf{v} \bullet \mathbf{w} = 4\,|\mathbf{w}|^2$.

2. In this problem, **n**, **m**, and **p** are non-zero vectors in $\mathbb{R}^3$ that are at right angles to each other. To answer Parts a), b) and c), put T next to the statement if it is true and F if it is false. There is no need to justify your answer for these parts. Part d) is <u>not</u> a true-false question and requires a justified answer.
   a) If **q** is a vector with **q**•**n** = 0, then **q** must lie in the plane through the origin whose normal vector is **n**.
   b) There is a vector **q** not proportional to **p** with both **q**•**n** = 0 and **q**•**m** = 0.
   c) If **q** is a vector that is orthogonal to **n**, **m**, and **p** simultaneously, then **q** must be zero.

   d) You are now told that **n** = (-2, 1, -3) but told nothing about **m** and **p** except that they are non-zero and at right angles to **n** and to each other. Even so, give an equation that is obeyed by all points (x, y, z) in the plane that contains the three points (1, 0, 0), (1, 0, 0) + **m**, and (1, 0, 0) + **p**.

3. A curve in $\mathbb{R}^2$ is parameterized as $t \to \mathbf{q}(t) = \frac{1}{\sqrt{2}}(\cos(t)\, e^t, \sin(t)\, e^t)$ with t between 0 and 1.
   a) Write down the velocity vector, $\mathbf{q}'(t)$, as a function of time.
   b) Find the length of the path traced out by **q**(t).

4. A path in $\mathbb{R}^2$ is parameterized via polar coordinates as $t \to (r(t), \theta(t))$, where $r(t) = 8 + \sin(t)$ and $\theta(t) = t$ where $-\infty < t < \infty$. Determine all times t where a particle moving in the plane via this parameterization has velocity vector that is parallel to the x-axis.

5. A line L in $\mathbb{R}^3$ is parameterized via $t \to (1 + 4t, 2t, -4t)$.
   a) Determine the equation of the plane that contains both L and the origin.
   b) Determine the distance from the point (1, 1, 1) to L.
   c) Determine the distance from the point (1, 1, 1) to the plane from Part a).

6. Here is Wile E. Coyote's latest scheme to catch Road Runner: For time $t < -10$, Coyote watches Road Runner from a high vantage point and determines that Road Runner is travelling in the x-y plane along flat ground, the z = 0 plane in $\mathbb{R}^3$. Moreover, Coyote sees that Road Runner's path is parameterized by time t via $t \to (2\sin(\pi t), -2t, 0)$. In the hope of landing on Road Runner, Coyote then starts at $t = -10$ along the trajectory parameterized via $t \to (t, -2, -100t - 5t^2)$ so as to hit the ground (where z = 0) at t=0 and, with luck, grab Road Runner.
   a) What are Coyote's coordinates at the launch site?
   b) How far does Coyote land from Road Runner? Does Coyote land on Road Runner?
   c) How fast is Coyote travelling when the ground is hit?
   d) What is the cosine of the angle between Coyote's velocity vector and the positive z-axis at the point where Coyote hits the ground.