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Math 21a Exam #1: Wednesday, March 7, 2001

SECTION (CIRCLE ONE):

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Question	Points	Score
1	16	
2	16	
3	16	
4	16	
5	18	
6	18	
Total	100	

The time allotted for this exam is 90 minutes.

Justify your answers carefully. No partial credit can be given for unsubstantiated answers.

If more space is needed, use the back of the previous page and make note of this.

Please write neatly. Answers which are deemed illegible by the grader will not receive credit.

No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students.

(1) Given the following points: $P(1, 2, -2)$, $Q(-3, 1, -14)$, $R(-1, 2, -6)$, and $S(1, -2, -8)$. Are these points coplanar, i.e. do they all lie in a common plane in \mathbf{R}^3 ? Justify your answer.

(2) Use vectors to show that the diagonals of a parallelogram are perpendicular if and only if all four sides of the parallelogram have equal length. (*Such a figure is called a rhombus.*)

- (3) Consider a pyramid-shaped tent of height 5 meters and with a 4 meter by 4 meter square base.
- (a) The top of the tent lies directly over the center of the base. Find the angle between two adjacent triangular faces of the tent, i.e. the angle between their outward pointing normal vectors.

- (b) If the tent is made of canvas and consists of the four triangular sides plus the square bottom, find the total area of canvas used to make the tent.

(4) Consider the vector field $\mathbf{F}(x, y) = (y + 2x)\mathbf{i} + (x - 3)\mathbf{j}$. We wish to find the work integral $\int_{\gamma} \mathbf{F} \cdot d\mathbf{x}$ along the path γ , where γ is the line segment from the point $(0, 2)$ to the point $(4, -2)$.

(a) Sketch the direction of \mathbf{F} at the five points $(0, 2)$, $(1, 1)$, $(2, 0)$, $(3, -1)$, and $(4, -2)$. On the basis of this sketch, explain whether you expect the value of the integral to be positive, negative, or zero.

(b) Evaluate the integral $\int_{\gamma} \mathbf{F} \cdot d\mathbf{x}$ along the line segment from the point $(0, 2)$ to $(4, -2)$.

(5) Consider the function $f(x, y) = \sqrt{50 - x^2 - y^2}$.

(a) Describe geometrically the surface formed by the graph $z = f(x, y)$.

(b) Sketch and label the contour of the function f through the point $(x, y) = (3, 4)$.

(c) Calculate ∇f , the gradient of f , at the point $(x, y) = (3, 4)$ and indicate it on your diagram above.

(d) Calculate the directional derivative of f at the point $(3, 4)$ in the direction of the vector $(2, -1)$.

(e) Find an equation for the plane tangent to the graph $z = f(x, y)$ at the point on the graph where $x = 3$ and $y = 4$.

(6) The Texahoma Saloon is situated on the Texas-Oklahoma border. Its roof is perpendicular to the vector $(1, -2, 2)$ and includes the point with coordinates $(-2, 4, 5)$. [The x -axis runs from west to east, the y -axis from south to north, and the z -axis points vertically upward.] Marshall Wyatt Zorn has been sent from Minnesota to this border country to enforce a recent executive order decreeing that all distances are to be measured in meters.

(a) Write an equation satisfied by the coordinates (x, y, z) of any point on the roof.

(b) The state line lies along the plane $y = 4$ (with $y > 4$ in Oklahoma and $y < 4$ in Texas, of course). Marshall Zorn is asked to paint the state line on the saloon's roof. Find parametric equations for this line. (You do not have to specify a parameter range.)

(c) Prominently posted in the saloon is a sign that states "No firearms within 2 meters of the roof." Calamity W. Jane, while trying to make a point about the Second Amendment, holds her Colt 45 at the point $(-3, -3, 2)$. Marshall Zorn needs to decide whether she is in violation of the law. Help him out by calculating the distance from the Colt to the roof.

(d) In panic at the prospect of being arrested, Jane fires her Colt 45. The bullet travels with great velocity proportional to the vector $(5, 6, 2)$ and moves in a straight line through the roof. Determine the location of the hole that it makes.