

Probability Homework 2 (Discrete Random Variables)
Biochem 21a Fall 2004

1. Consider the experiment of flipping a coin three times, and write down the eight sequences of heads and tails in the outcome space. Let the random variable X be the number of heads in the three tosses, let the random variable Y be the number of tails, and let the random variable Z have value 1 if the first toss is heads, and value 0 if the first toss is tails.

(a) Draw the distributions of X , Y , and Z . Are any of the distributions the same? Are any of X , Y , and Z equal?

(b) Draw the conditional distributions of Y given $X = x$ for all x in the range of X . Are X and Y independent?

(c) Draw the conditional distributions of Y given $Z = z$ for all z in the range of Z .

(d) Completely describe the joint outcome (X, Y) . That is, determine the range of (X, Y) , defined to be the set of points (x, y) where x is in the range of X , y is in the range of Y , and $P(X = x, Y = y) \geq 0$. Then, for each point (x, y) in the range of (X, Y) , determine $P(X = x, Y = y)$.

(e) As in (d), completely describe the joint outcome (X, Y, Z) .

2. Suppose a bag contains 3 red balls and 4 black balls. Three balls are chosen without replacement. Let the random variable X be the number of red balls chosen.

(a) Completely describe X , giving $P(X = x)$ for every x in the range of X , and draw the distribution.

(b) Calculate

- $P(1 \leq X < 3)$
- $P(1 < X \leq 3)$
- $P(1 < X < 3)$
- $P(1 \leq X < 3 | X < 2)$

(c) Find $E(X)$ and $Var(X)$.

3. Marta is a health worker who contracts SARS at the beginning of day 1 and is highly infectious. While she isn't quarantined, she comes in contact with eight people each day, none of whom she ever sees again. While she has SARS and is not quarantined, she has a 25% chance of infecting each person she contacts. At the end of each day, she has a 10% chance of getting better and no longer being infectious, and a 15% chance of getting so sick that she is diagnosed and quarantined so that she can no longer infect people. Let the random variable X be the number of days Marta is infectious and not quarantined, let the random variable Y_i be the number of people she infects on day i , and let the random variable Y be the total number of people she infects.

(a) Compute $P(X \geq x)$ for every x in the range of X .

(b) Completely describe X , giving $P(X = x)$ for every x in the range of X . Check that $P(X = 1) + P(X = 2) + P(X = 3) + \dots$ equals 1. Draw the distribution of X . Compute $E(X)$.

(c) Draw the conditional distribution of Y_i given $X \geq i$. Compute $E(Y_i | X \geq i)$.

(d) Completely describe Y_i as in (b), and draw its distribution. Compute $E(Y_i)$.

(e) Compute $E(Y)$. (Approximation to three significant figures is okay if you don't want to give an exact fraction.) Is $E(Y) = E(X)E(Y_1)$? Why should or shouldn't this be the case?

4. Suppose we have 2 urns. Urn I contains 2 white and 3 red balls and urn II contains 1 white and 4 red balls. A ball is selected simultaneously and at random from each urn and placed in the other urn. Then 1 ball is selected from each urn. Let X be the number of white balls drawn during the second selection process. Find $E(X)$ and $Var(X)$.