Review Exercises

1. A factory produces items in boxes of 2. Over the long run:

92% of boxes contain 0 defective items;

5% of boxes contain 1 defective item; and

3% of boxes contain 2 defective items.

A box is picked at random from production, then an item is picked at random from the box. Given that the item is defective, what is the chance that the second item in the box is defective?

2. A box contains 1 black ball and 1 white ball. A ball is drawn at random, then replaced in the box with an additional ball of the same color. Then a second ball is drawn at random from the three balls in the box. What is the probability that the first ball drawn was white, given that at least one of the two balls drawn was white?

3. Suppose I toss three coins. Two of them at least must land the same way. No matter whether they land heads or tails, the third coin is equally likely to land either the same way or oppositely. So the chance that all three coins land the same way is $1/2$. True or False? Explain!

4. There are two boxes.
Box 1 contains 2 red balls and 3 black balls.
Box 2 contains 8 red balls and 12 black balls.
One of the two boxes is picked at random, and then a ball is picked at random from the box.

a) Is the color of the ball independent of which box is chosen?

b) What if there were 10 black balls rather than 12 in Box 2, but the other numbers were the same?

5. To pass a test you have to perform successfully two consecutive tasks, one easy and one hard. The easy task you think you can perform with probability z, and the hard task you think you can perform with probability h, where $h < z$. You are allowed three attempts, either in the order (easy, hard, easy) or in the order (hard, easy, hard). Whichever order, you must be successful twice in a row to pass. Assuming that your attempts are independent, in what order should you choose to take the tasks in order to maximize your probability of passing the test?

6. Show that if A and B are independent, then so are A^c and B, A and B^c, and A^c and B^c.

7. A population of 50 registered voters contains 30 in favor of Proposition 134 and 20 opposed. An opinion survey selects a random sample of 4 voters from this population, as follows. One person is picked at random from the 50 voters, then another at random from the remaining 49, and so on, till 4 people have been picked.

a) What is the probability that there will be no one in favor of 134 in the sample?

b) What is the probability that there will be at least one person in favor?

c) What is the probability that exactly one pro 134 person will appear in the sample?

d) What is the probability that the majority of the sample will be pro 134? (Majority means strictly more than half.)

8. Cards are dealt from a well-shuffled standard deck until the first heart appears.

a) What is the probability that exactly 5 deals are required?

b) What is the probability that 5 or fewer deals are required?

c) What is the probability that exactly 3 deals were required, given that 5 or fewer were required?

9. Suppose events A, B, and C are independent with probabilities $1/5$, $1/4$, and $1/3$, respectively. Write down numerical expressions for the following probabilities:

a) $P(A \text{ and } B \text{ and } C)$

b) $P(A \text{ or } B \text{ or } C)$

c) $P(\text{exactly one of the three events occurs})$

10. The four major blood types are present in approximately the following proportions in the population of the U.S.A.

Type	A	B	AB	0
proportion	42%	10%	4%	44%

Note that each person's blood is exactly one of these four types. Type AB is a separate type, not the intersection of type A and type B.

a) If two people are picked at random from this population, what is the chance that their blood is of the same type? Of different types?

b) If four people are picked at random, let $P(k)$ be the chance that there are exactly k different blood types among them. Find $P(k)$ for $k = 1, 2, 3, 4$.

11. A hat contains n coins, f of which are fair, and b of which are biased to land heads with probability $2/3$. A coin is drawn from the hat and tossed twice. The first time it lands heads, and the second time it lands tails. Given this information, what is the probability that it is a fair coin?

12. Suppose n ordinary dice are rolled.

a) What is the chance that the dice show n different faces?

b) What is the chance that at least one number appears more than once?

13. **Formula for $P(A \mid B)$ by conditioning on cases of B.** Show if $B_1, \ldots, B_n$ is a partition of B, then

$$P(A|B) = P(A|B_1)P(B_1|B) + \cdots + P(A|B_n)P(B_n|B)$$

14. There are 100 boxes, and for each $i = 1, 2, \ldots, 100$, box i contains proportion $i/100$ of gold coins (the rest are silver). One box is chosen at random, then a coin is drawn at random from this box.

a) If the coin drawn is gold, which box would you guess was chosen? Why?

b) Suppose the boxes were not picked at random, but according to the following scheme. All the even-numbered boxes are equally likely, all the odd-numbered boxes are equally likely, but the chance of drawing an odd-numbered box is twice the chance of drawing an even-numbered box. If the coin drawn is gold, which box would you guess was chosen? [*Hint:* Write down the prior odds.]

15. There are three boxes, each with two drawers. Box 1 has a gold coin in each drawer, and box 2 has a silver coin in each drawer. Box 3 has a silver coin in one drawer and a gold coin in the other. One box is chosen at random, then a drawer is chosen at random from the box. Find the probability that box 1 is chosen, given that the chosen drawer yields a gold coin.

16. A dormitory has n students, all of whom like to gossip. One of the students hears a rumor, and tells it to one of the other $n-1$ students picked at random. Subsequently, each student who hears the rumor tells it to a student picked at random from the dormitory (excluding, of course, himself/herself and the person from whom he/she heard the rumor). Let p_r be the probability that the rumor is told r times without coming back to a student who has already heard it from a dormitory-mate. So $p_1 = p_2 = 1$, and $p_n = 0$.

a) Find a formula for p_r for r between 3 and $n-1$.

b) Estimate this probability for $n = 300$ and $r = 30$.

17. Some time ago I received the following letter:

"You may have previously received a letter notifying you that you had been a selectee in a recent sweepstake that we were conducting. According to our records, you have not claimed your gift.
We are always pleased when our bigger gifts are awarded because it's good publicity for our company. However, last year there were thousands of dollars worth of unclaimed gifts simply because the selectees failed to respond.
This letter is to inform you that one of the following people has won a New Datsun Sentra:

Collin Andrus	Oklahoma City, OK
James W. Pitman	Berkeley, CA
Larry Abbott	Burbank, CA

In compliance with the rules of the sweepstake, you are hereby notified that you are a selectee in Category I, which means you will receive one of the following:

1. R.C.A. Color TV;
2. 5 FT. Grandfather Clock;
3. Datsun Nissan Sentra.

To claim your gift, all you have to do is call toll free 1-800-643-3249 for an available time and date for you and your spouse to visit Heavenly Valley Townhouses and attend a sales representation tour on the many advantages that interval ownership has to offer."

According to small print on the back of the letter:

"The retail values and odds of receiving each gift are No. 1—1/10,000 ($329.95), No. 2—9998/10,000 ($249.95), No. 3—1/10,000($5,995.00)."

Let us assume that this is an honestly conducted sweepstake, and that each of the three individuals named above had originally a 1 in 10,000 chance of winning the new Datsun. Now I know that the winner is one of these individuals, the rules of conditional probability imply that I have a one in three chance of winning the Datsun. True or false? Explain!

18. Suppose there are m equally likely possibilities for one stage, and n equally likely possibilities for another. Show that the two stages are independent if and only if all mn possible joint outcomes are equally likely.

19. A box contains 5 tickets numbered $1, 2, 3, 4$, and 5. Two tickets are drawn at random from the box. Find the chance that the numbers on the two tickets differ by two or more if the draws are made:

a) with replacement;

b) without replacement.

Repeat the problem with n tickets numbered $1, 2, \ldots, n$.

1.rev.11. $\frac{9f}{9f+8b}$

1.rev.13. *Hint:* Write $P(A|B) = \sum_{i=1}^{n} P(AB_i|B)$.

1.rev.15. 2/3

1.rev.17. False.

1.rev.1. 6/11

1.rev.3. False

1.rev.5. The chance of passing when you use the first order is $zh(2-z)$. With the second order, it's $hz(2-h)$.

1.rev.7. a) $\frac{20}{50} \cdot \frac{19}{49} \cdot \frac{18}{48} \cdot \frac{17}{47} = .021$ b) 1 − answer to a) c) $4 \cdot \frac{30}{50} \cdot \frac{20}{49} \cdot \frac{19}{48} \cdot \frac{18}{47}$
d) $4 \cdot \frac{30}{50} \cdot \frac{29}{49} \cdot \frac{28}{48} \cdot \frac{20}{47} + \frac{30}{50} \cdot \frac{29}{49} \cdot \frac{28}{48} \cdot \frac{27}{47}$

1.rev.9. a) 1/60 b) 3/5 c) 13/30