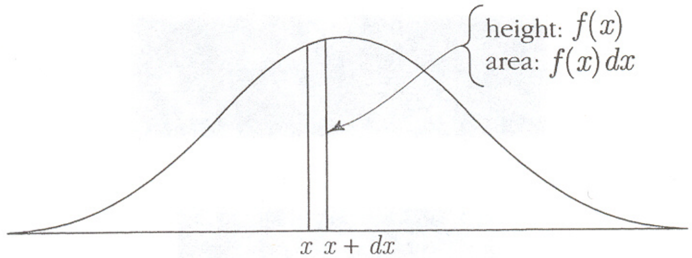
Distributions Defined by a Density

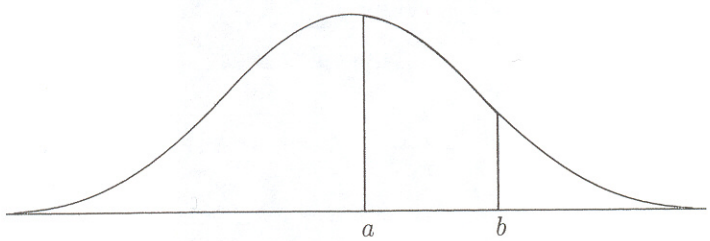
Infinitesimal Probability:



$$P(X \in dx) = f(x)dx$$

The *density* $f(x)$ gives the probability per unit length for values near x.

Interval Probability:



$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

the area under the graph of $f(x)$ between a and b.

Constraints: Non-negative with Total Integral 1

$$f(x) \geq 0 \quad \text{for all } x \qquad \text{and} \qquad \int_{-\infty}^{\infty} f(x)dx = 1$$

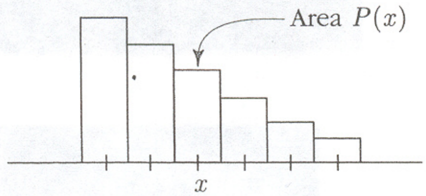
Expectation of a Function g of X, e.g., X, X^2:

$$E\,(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)dx$$

provided the integral converges absolutely.

Discrete Distributions

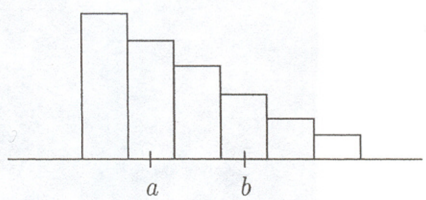
Point Probability:



$$P(X = x) = P(x)$$

So $P(x)$ is the probability that X has integer value x.

Interval Probability:



$$P(a \le X \le b) = \sum_{a \le x \le b} P(x)$$

the relative area under a histogram between $a - 1/2$ and $b + 1/2$.

Constraints: Non-negative with Total Sum 1

$$P(x) \ge 0 \quad \text{for all } x \qquad \text{and} \qquad \sum_{\text{all } x} P(x) = 1$$

Expectation of a Function g of X, e.g., X, X^2:

$$E\left(g(X)\right) = \sum_{\text{all } x} g(x)P(x)$$

provided the sum converges absolutely.

Binomial Distribution

For n independent trials, with probability p of success and probability $q = 1-p$ of failure on each trial, the probability of k successes is given by the *binomial probability formula*:

$$P(k \text{ successes in } n \text{ trials}) = \binom{n}{k} p^k q^{n-k}$$

where $\binom{n}{k}$, called *n choose k*, is the number of different possible patterns of k successes and $n-k$ failures in n trials, given by the formula

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1} = \frac{n!}{k!(n-k)!}$$

Here the $k!$ is *k factorial*, the product of the first k integers for $k \geq 1$, and $0! = 1$. For fixed n and p, as k varies, these binomial probabilities define a probability distribution over the set of $n+1$ integers $\{0, 1, \ldots, n\}$, called the *binomial (n, p) distribution*. This is the distribution of the number of successes in n independent trials, with probability p of success in each trial. The binomial (n, p) probabilities are the terms in the *binomial expansion*:

$$(p+q)^n = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k}$$

Appendix 1 gives the background on counting and a derivation of the formula fc $\binom{n}{k}$ in the box. The first expression for $\binom{n}{k}$ in the box is the simplest to use fc