Last Name: ______________________________

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Math 21a Exam #2: Tuesday, April 4, 2000

SECTION (CIRCLE ONE):

Robert Winters
John Friedman (CA)
MWF 10-11

Marty Weissman
Lukasz Fidkowski (CA)
MWF 11-12

Tom Weston
Nick Stang (CA)
MWF 11-12

Russell Mann
Mike Grobis (CA)
MWF 11-12 (Physics)

Tom Weston
James Griffin (CA)
MWF 12-1 (BioChem)

Yang Liu
Bartok Czech (CA)
TTh 10-11:30

Daniel Allcock
Nikhil Dutta (CA)
TTh 11:30-1

Question	Points	Score
1	16	
2	18	
3	16	
4	17	
5	17	
6	16	
Total	100	

The time allotted for this exam is 90 minutes.

Justify your answers carefully. No partial credit can be given for unsubstantiated answers.

If more space is needed, use the back of the previous page and make note of this.

Please write neatly. Answers which are deemed illegible by the grader will not receive credit.

No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students.

In agreeing to take this exam, you are implicitly accepting Harvard University's Honor Code.

(1) Find all critical points of the function

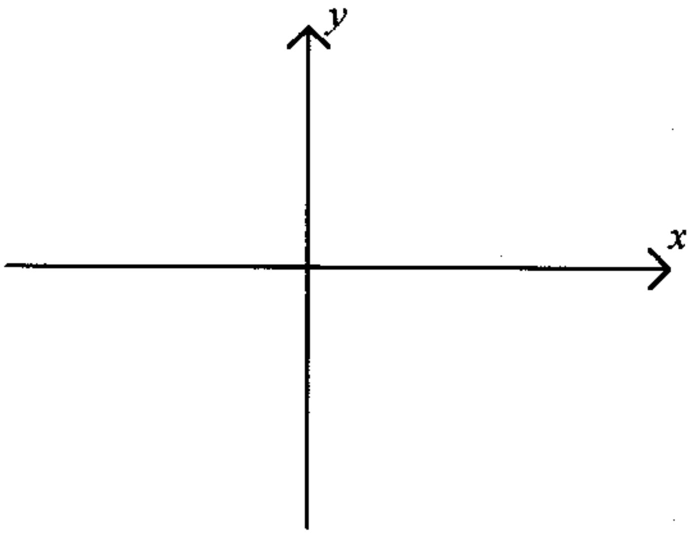
$$f(x, y) = 8x^3 + 6xy - 3y^2 - 24x - 6y + 5$$

Classify each critical point as a local minimum, a local maximum, or a saddle point.

(2) Let $f(x, y) = x^2 - 2y$.

a) Draw several contours (level curves) for this function, and label these curves with the values corresponding to each curve. In particular, draw the contour passing through the point (2, 1).

b) Calculate the gradient ∇f. Illustrate on your level curve diagram the direction of the gradient vector at various points. In particular, find $\nabla f(2, 1)$ and show its direction on the appropriate level curve.



c) Find the directional derivative of f at the point (2, 1) in the direction given by the vector $\mathbf{v} = (-2, 3)$. Are the values of the given function increasing or decreasing in this direction?

d) Suppose you move along the curve given parametrically by:

$$x(t) = 2e^{2t}, \quad y(t) = 2t^3 + 6t + 1.$$

What will $\frac{df}{dt}$ be as you travel along this path when $t = 0$?

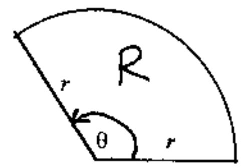
(3) Let $f(x,y)=\sqrt{x^4+2xy^2}$.

(a) What is the (best) linear approximation $L(x, y)$ of this function at the point (1, 2)?

(b) Use the approximation in (a) to estimate $f(0.9, 2.03)$.

(4) Find the point on the plane $2x + 2y - z = 15$ closest to the point $(-1,1,3)$ using the Method of Lagrange Multipliers.

(5) A region R is given by a piece of a disk of radius r as shown. If the total perimeter of R is equal to 120 centimeters, what values of r and θ will maximize the area of R? What is this maximum area?



(6) [Regular and BioChem sections]
Consider the integral

$$\int_0^6 \int_{x/3}^2 x\sqrt{y^3+1}\,dy\,dx \ .$$

(a) Sketch the region of integration.

(b) Evaluate the integral.

(6) [Physics sections] Compute the line integral $\int_\gamma \mathbf{F}\cdot\mathbf{dx}$ in the following cases:

(a) γ is the line segment in $\mathbf{R}^2$ from (0, 1) to (3, 5), and $\mathbf{F}(x, y) = (x^2 - y, xy + 1)$

(b) γ is the path in $\mathbf{R}^3$ from (1, 0, 0) to (1, 0, 2π) along the helix given parametrically by
$\mathbf{x}(t) = (\cos t, \sin t, t)$, and
$\mathbf{F}(x, y, z) = (2x + 2y - 3z, 2x + z + 4, -2z - 3x + y)$