

Answers to Math 21a Hourly 1
(Fall 2001)

1. a) F b) T c) T d) T e) T f) F
2. a) T or F depending on your interpretation. b) F c) T
d) $2x + y - 3z = 2$. Alternately, you can $\Pi = (1, 0, 0) + t \mathbf{a} + s \mathbf{b}$ where t and s are real numbers and \mathbf{a} and \mathbf{b} are non-zero vectors that have zero dot product with \mathbf{n} .

Note that a)-c) counted 2 points each while d) counted for 4 points.

3. a) $\mathbf{q}'(t) = \frac{1}{\sqrt{2}} (-\sin(t) e^t + \cos(t) e^t, \cos(t) e^t + \sin(t) e^t)$.
- b) As $|\mathbf{q}'(t)| = e^t$, the length of the path is $\int_0^1 e^t dt = e - 1$.
4. The answer is $t = (\frac{1}{2} + n) \pi$, where n is any integer.
Solution 1: Find that $y'(t) = 2 \cos(t) (\sin(t) + 4)$, and so vanishes only where $\cos(t)$ does. In this regard, $x(t) = (8 + \sin(t)) \cos(t)$ and $y(t) = (8 + \sin(t)) \sin(t)$.
Solution 2: Draw the graph.

5. a) If $Q=(1,0,0)$, $P=(0,0,0)$, then $P-Q$ is in the plane as well as $v=(4,2,-4)$.
The vector $n=(P-Q) \times v=(0,4,2)$ is normal to the plane so that $4y+2z=d$ is the equation of the plane. Because $(0,0,0)$ is on the plane, $4y+2z=0$.
- b) Use the formula $|v \times (P-Q)|/|v|=\text{sqrt}(17)/3$
- c) Use the formula $\ln \cdot (P-Q)/|n|=3/\text{sqrt}\{5\}$
6. a) (-10, -2, 500)
- b) Coyote hits the ground at $t = 0$. At this point, Coyote's coordinates are (0, -2, 0) while Road Runner's are (0, 0, 0). Thus, Coyote is at distance 2 from Road Runner.
- c) Coyote's velocity vector is (1, 0, -100 - 10 t), so equals (1, 0, -100) at $t = 0$ when the ground is hit. His speed is the length of this vector, $(10,001)^{1/2}$.
- d) The cosine of the angle is the ratio of the dot product of the velocity vector with the unit vector (0, 0, 1). Thus, the cosine is $-100 (10,001)^{-1/2}$.