For each of the following problems, find the range of the random variable X and, for each y in the range of X, find the indicated probability.

1) Toss a coin 4 times. Let X be the number of heads minus the number of tails. Find $P(X = y)$.

2) Toss a pair of dice. Let X be the sum of the numbers facing up on the dice. Find $P(X = y)$.

3) Toss a die 5 times. Let X be the number of 6's that appear. Find $P(X = y)$.

4) Let $S = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$. If (a,b) is a randomly chosen point in S, let X be the distance from the origin to (a,b). Find $P(X \leq t)$ for each t in the range of X.

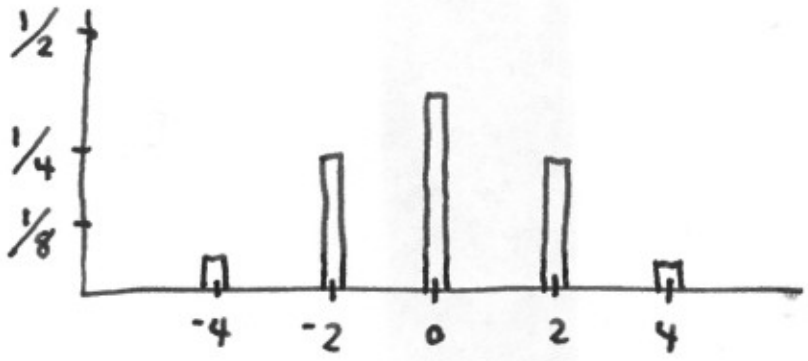
Work Sheet solutions

1) Range $X = \{-4, -2, 0, 2, 4\}$

$P(X = -4) = 1/16 = P(X = 4)$

$P(X = -2) = 1/4 = P(X = 2)$

$P(X = 0) = 3/8$



2) Range $X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$P(X = 2) = P(X = 12) = 1/36$

$P(X = 3) = P(X = 11) = 2/36 = 1/18$

$P(X = 4) = P(X = 10) = 3/36 = 1/12$

$P(X = 5) = P(X = 9) = 4/36 = 1/9$

$P(X = 6) = P(X = 8) = 5/36$

$P(X = 7) = 6/36 = 1/6$

3) $\text{Range}(X) = \{0, 1, 2, 3, 4, 5\}$

$$P(X=0) = \binom{5}{0}\left(\frac{5}{6}\right)^5 \approx 0.4019$$

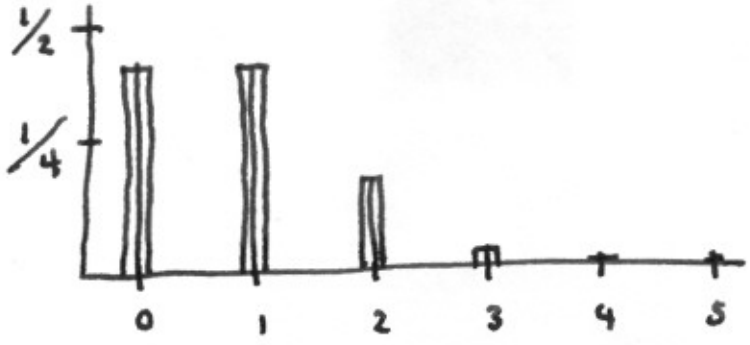
$$P(X=1) = \binom{5}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4 \approx 0.4019$$

$$P(X=2) = \binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 \approx 0.1608$$

$$P(X=3) = \binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 \approx 0.0322$$

$$P(X=4) = \binom{5}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right) \approx 0.0032$$

$$P(X=5) = \binom{5}{5}\left(\frac{1}{6}\right)^5 \approx 0.00013$$



4) $\text{Range}(X) = \{x \mid 0 \le x \le 1\}$

$$P(X \le t) = \frac{\pi t^2}{\pi} = t^2$$