PRACTICE EXAM FIRST HOURLY **Math 21a, Fall 2005**

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- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

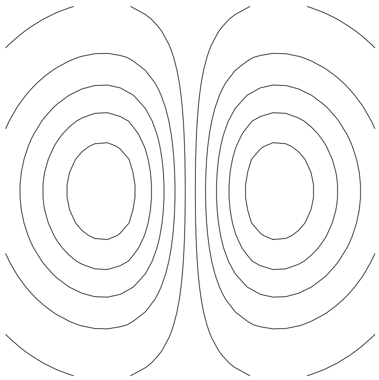
Problem 1) TF questions (20 points) No justifications needed

1)	T	F	The length of the sum of two vectors is always the sum of the length of the vectors.
2)	T	F	For any three vectors, $\vec{v} \times (\vec{w} + \vec{u}) = \vec{w} \times \vec{v} + \vec{u} \times \vec{v}$.
3)	T	F	The set of points which satisfy $x^2 + 2x + y^2 - z^2 = 0$ is a cone.
4)	T	F	The functions $\sqrt{x+y-1}$ and $\log(x+y-1)$ have the same domain of definition.
5)	T	F	If P, Q, R are 3 different points in space that don't lie in a line, then $\vec{PQ} \times \vec{RQ}$ is a vector orthogonal to the plane containing P, Q, R.
6)	T	F	The line $\vec{r}(t) = (1+2t, 1+3t, 1+4t)$ hits the plane $2x + 3y + 4z = 9$ at a right angle.
7)	T	F	The graph of $f(x,y) = \cos(xy)$ is a level surface of a function $g(x,y,z)$.
8)	T	F	For any two vectors, $\vec{v} \times \vec{w} = \vec{w} \times \vec{v}$.
9)	T	F	If $\lvert\vec{v} \times \vec{w}\rvert = 0$ for all vectors $\vec{w}$, then $\vec{v} = \vec{0}$.
10)	T	F	If $\vec{u}$ and $\vec{v}$ are orthogonal vectors, then $(\vec{u} \times \vec{v}) \times \vec{u}$ is parallel to $\vec{v}$.
11)	T	F	Every vector contained in the line $\vec{r}(t) = \langle 1+2t, 1+3t, 1+4t \rangle$ is parallel to the vector $\langle 1,1,1 \rangle$.
12)	T	F	The curvature of the curve $2\vec{r}(4t)$ at $t = 0$ is twice the curvature of the curve $\vec{r}(t)$ at $t = 0$.
13)	T	F	The set of points which satisfy $x^2 - 2y^2 - 3z^2 = 0$ form an ellipsoid.
14)	T	F	If $\vec{v} \times \vec{w} = (0,0,0)$, then $\vec{v} = \vec{w}$.
15)	T	F	The set of points in space which have distance 1 from a line form a cylinder.
16)	T	F	Every vector contained in the line $\vec{r}(t) = \langle 1+2t, 1+3t, 1+4t \rangle$ is parallel to the vector $\langle 1,1,1 \rangle$.
17)	T	F	The equation $x^2 + y^2/4 = 1$ in space describes an ellipsoid.
18)	T	F	For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, we always have $(\vec{a} \times \vec{b}) \cdot \vec{c} = -(\vec{a} \times \vec{c}) \cdot \vec{b}$.
19)	T	F	The set of points in the xy-plane which satisfy $x^2 - y^2 = -1$ is a hyperbola.
20)	T	F	Two nonzero vectors are parallel if and only if their cross product is $\vec{0}$.

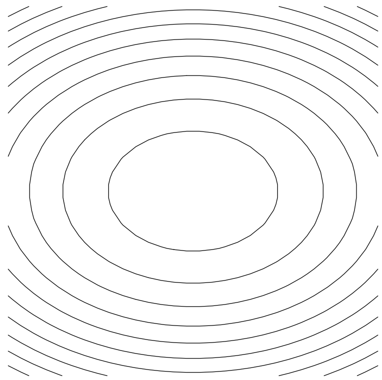
Problem 2) (10 points)

Match the contour maps with the corresponding functions $f(x, y)$ of two variables. No justifications are needed.

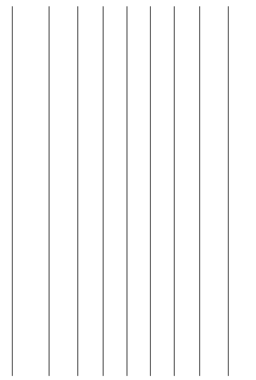
I



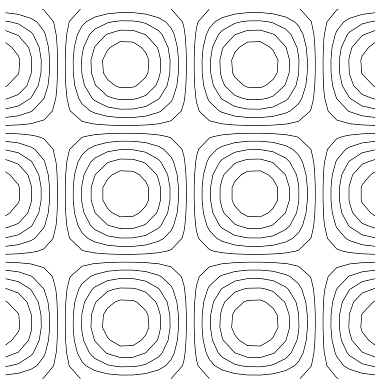
II



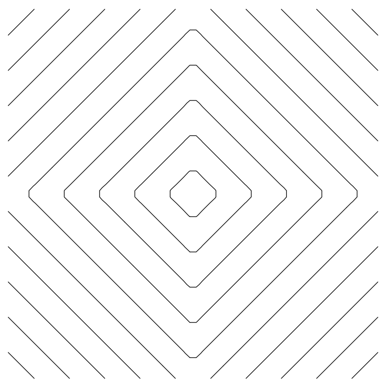
III



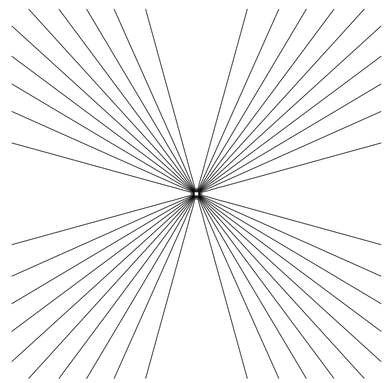
IV



V



VI



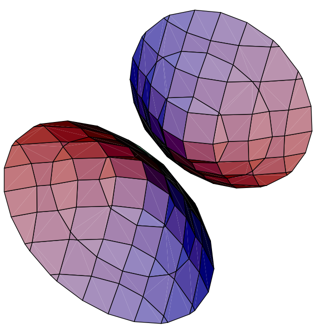
Non-graphical description:

I) shows level curves which are symmetric with respect to the y axes, on both sides, there are concentric lense shaped closed curves.
II) shows concentric elliptic level curves
III) shows a family of vertical lines
IV) shows many a periodic pattern of level curves, where each pattern contains in the center circle like curves which become more and more square like
V) shows diamond shaped level curves with corners
VI) shows a family of lines which meet in the center.

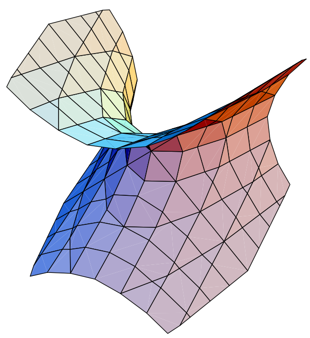
Enter I,II,III,IV,V or VI here	Function $f(x,y)$
	$f(x,y) = \sin(x)$
	$f(x,y) = x^2 + 2y^2$
	$f(x,y) = \|x\| + \|y\|$
	$f(x,y) = \sin(x)\cos(y)$
	$f(x,y) = xe^{-x^2-y^2}$
	$f(x,y) = x^2/(x^2+y^2)$

Problem 3) (10 points)

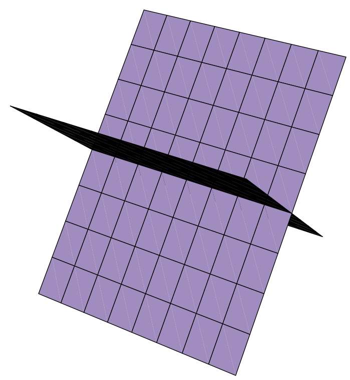
Match the equation with the pictures and justify briefly your choice.



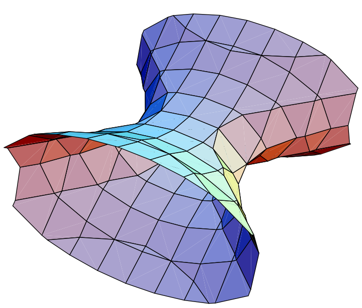
I



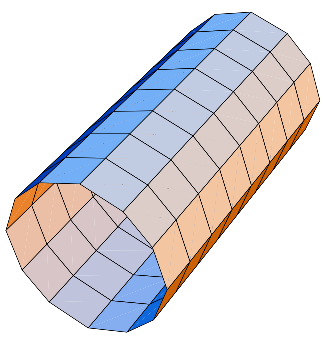
II



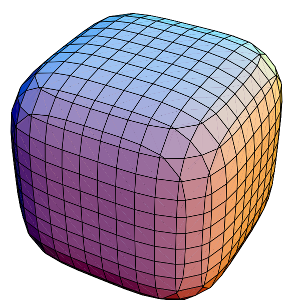
III



IV



V



VI

Enter I,II,III,IV,V,VI here	Equation	Short explanation
	$x^4 + y^4 + z^4 - 1 = 0$	
	$-x^2 + y^2 - z^2 - 1 = 0$	
	$x^2 + z^2 = 1$	
	$-y^2 + z^2 = 0$	
	$x^2 - y^2 + 3z^2 - 1 = 0$	
	$x^2 - y - z^2 = 0$	

Nongraphical description:
Figure I) shows two shells, where each shell looks like a bowl
Figure II) shows a surface which looks like the front part of a human neck cut on the top below the chin and at the bottom at the upper part of the shoulder
Figure III) shows two planes which cross each other orthogonally
Figure IV) shows a tube like surface which is narrow in the middle, and wide at the ends and which has circular cross sections
Figure V) shows a tube like surface with circular cross section
Figure VI) shows a rounded cube

Problem 4) (10 points)

a) (6 points) Find a parameterization of the line of intersection of the planes $3x - 2y + z = 7$ and $x + 2y + 3z = -3$.

b) (4 points) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing that line.

Problem 5) (10 points)

a) (4 points) Find the area of the parallelogram with vertices $P = (1, 0, 0)$ $Q = (0, 2, 0)$, $R = (0, 0, 3)$ and $S = (-1, 2, 3)$.

b) (3 points) Verify that the triple scalar product has the property $[\vec{u}+\vec{v}, \vec{v}+\vec{w}, \vec{w}+\vec{u}] = 2[\vec{u}, \vec{v}, \vec{w}]$.

c) (3 points) Verify that the triple scalar product $[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$ has the property

$$|[\vec{u}, \vec{v}, \vec{w}]| \leq ||\vec{u}|| \cdot ||\vec{v}|| \cdot ||\vec{w}||$$

Problem 6) (10 points)

Find the distance between the two lines

$$\vec{r}_1(t) = \langle t, 2t, -t \rangle$$

and

$$\vec{r}_2(t) = \langle 1 + t, t, t \rangle \ .$$

Problem 7) (10 points)

Find an equation for the plane that passes through the origin and whose normal vector is parallel to the line of intersection of the planes $2x + y + z = 4$ and $x + 3y + z = 2$.

Problem 8) (10 points)

The intersection of the two surfaces $x^2 + \frac{y^2}{2} = 1$ and $z^2 + \frac{y^2}{2} = 1$ consists of two curves.

a) (4 points) Parameterize each curve in the form $\vec{r}(t) = (x(t), y(t), z(t))$.
b) (3 points) Set up the integral for the arc length of one of the curves.
c) (3 points) What is the arc length of this curve?

Problem 9) (10 points)

Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = \langle -\cos(t), \sin(t), -2t \rangle$.

Hint. Use one of the two formulas for the curvature

$$\kappa(t) = \frac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|} = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3} ,$$

where $\vec{T}(t) = \vec{r}\,'(t)/|\vec{r}\,'(t)|$.