Name:

MWF9 Ivan Petrakiev
MWF10 Oliver Knill
MWF10 Thomas Lam
MWF10 Michael Schein
MWF10 Teru Yoshida
MWF11 Andrew Dittmer
MWF11 Chen-Yu Chi
MWF12 Kathy Paur
TTh10 Valentino Tosatti
TTh11.5 Kai-Wen Lan
TTh11.5 Jeng-Daw Yu

- Please mark the box to the left which lists your section.
- Do not detach pages from this exam packet or unstaple the packet.
- Show your work! Answers without reasoning can not be given credit, except for the TF and multiple choice problems.
- Please write neatly. Answers which the grader can not read will not receive credit.
- No notes, books, calculators, computers, or other electronic aids can be used.
- All unspecified functions mentioned in this exam are assumed to be smooth: you can differentiate as many times as you want with respect to any variables.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) TF questions (20 points)

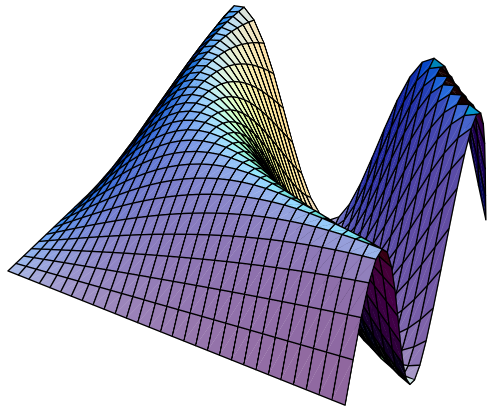
Mark for each of the 20 questions the correct letter. No justifications are needed.

1)	T	F	Any function of three variables $f(x,y,z)$ satisfies the partial differential equation $f_{xyz} + f_{yzx} = 2f_{zxy}$.
2)	T	F	If $f_x(x,y) = f_y(x,y)$ for all x, y, then $f(x,y)$ is a constant.
3)	T	F	$(1,1)$ is a local maximum of the function $f(x,y) = x^2y - x + \cos(y)$.
4)	T	F	If f is a smooth function of two variables, then the number of critical points of f inside the unit disc is finite.
5)	T	F	The value of the function $f(x,y) = \sin(-x+2y)$ at $(0.001, -0.002)$ can by linear approximation be estimated as -0.003.
6)	T	F	If $(1,1)$ is a critical point for the function $f(x,y)$ then $(1,1)$ is also a critical point for the function $g(x,y) = f(x^2, y^2)$.
7)	T	F	There is no function $f(x,y,z)$ of three variables, for which every point on the unit sphere is a critical point.
8)	T	F	If (x_0, y_0) is a maximum of $f(x,y)$ under the constraint $g(x,y) = g(x_0,y_0)$, then (x_0,y_0) is a maximum of $g(x,y)$ under the constraint $f(x,y) = f(x_0,y_0)$.
9)	T	F	If $\vec{u}$ is a unit vector tangent at (x,y,z) to the level surface of $f(x,y,z)$ then $D_uf(x,y,z) = 0$.
10)	T	F	In cylindrical coordinates (r,θ,z), the surface $z = r^2$ describes a cone.
11)	T	F	The function $u(x,t) = x^2/2 + t$ satisfies the heat equation $u_t = u_{xx}$.
12)	T	F	The vector $\vec{r}_u - \vec{r}_v$ is tangent to the surface parameterized by $\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$.
13)	T	F	If $(1,1,1)$ is a maximum of f under the constraints $g(x,y,z) = c, h(x,y,z) = d$, and the Lagrange multipliers satisfy $\lambda = 0, \mu = 0$, then $(1,1,1)$ is a critical point of f.
14)	T	F	If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D is zero but $f_{xx}(0,0) > 0$ then $(0,0)$ can not be a local maximum.
15)	T	F	Let (x_0,y_0) be a saddle point of $f(x,y)$. For any unit vector $\vec{u}$, there are points arbitrarily close to (x_0,y_0) for which ∇f is parallel to $\vec{u}$.
16)	T	F	A function $f(x,y)$ on the plane for which the absolute minimum and the absolute maximum are the same must be constant.
17)	T	F	The sign of the Lagrange multiplier tells whether the critical point of $f(x,y)$ constrained to $g(x,y) = 0$ is a local maximum or a local minimum.
18)	T	F	The gradient of a function $f(x,y,z)$ is tangent to the level surfaces of f
19)	T	F	The point $(0,1)$ is a local minimum of the function $x^3 + (\sin(y-1))^2$.
20)	T	F	If $D_uf(x,y,z) = 0$ for all unit vectors u, then (x,y,z) is a critical point.

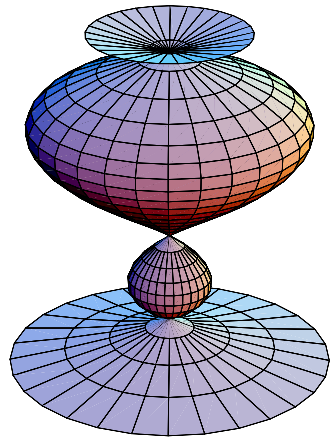
Space for work

Problem 2) (10 points)

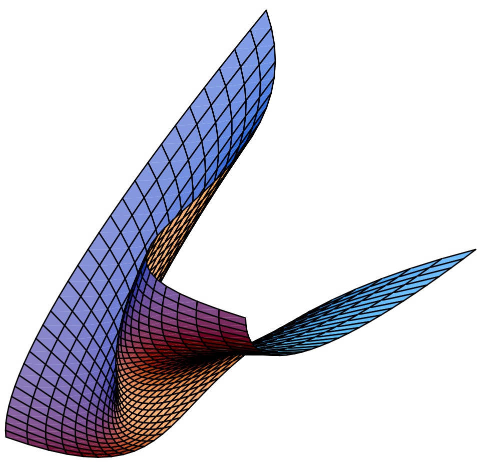
Match the parametric surfaces with their parameterization. No justification is needed.



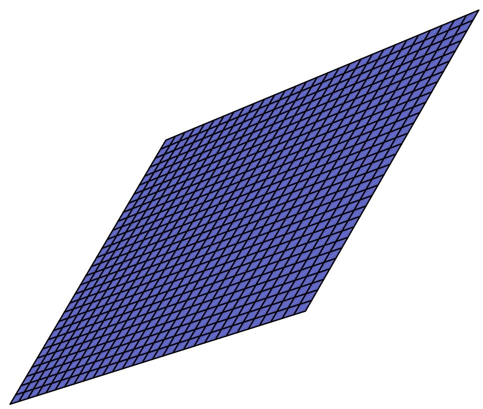
I



II



III



IV

Enter I,II,III,IV here	Parameterization
	$(u,v) \mapsto (u, v, u+v)$
	$(u,v) \mapsto (u, v, \sin(uv))$
	$(u,v) \mapsto (0.2 + u(1-u^2))\cos(v), (0.2 + u(1-u^2))\sin(v), u)$
	$(u,v) \mapsto (u^3, (u-v)^2, v)$

Space for work

Problem 3) (10 points)

Use the technique of linear approximation to estimate $f(\log(2)+0.001, 0.006)$ for $f(x,y) = e^{2x-y}$. (Here, log means the natural logarithm).

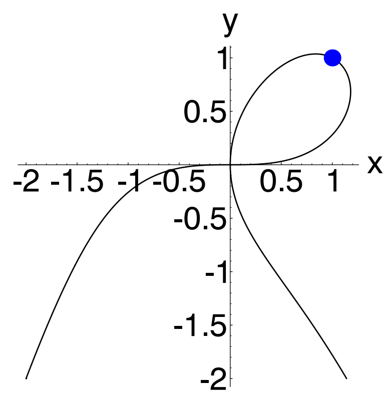
Space for work

Problem 4) (10 points)

Consider the equation

$$f(x, y) = 2y^3 + x^2y^2 - 4xy + x^4 = 0$$

It defines a curve, which you can see in the picture. Near the point $x = 1, y = 1$, the function can be written as a graph $y = y(x)$. Find the slope of that graph at the point $(1, 1)$.



Space for work

Problem 5) (10 points)

Find and classify all the critical points of the function $f(x, y) = xy(4 - x^2 - y^2)$.

Space for work

Problem 6) (10 points)

Let $f(x,y) = e^{(x-y)}$ so that $f(\log(2), \log(2)) = 1$. Find the equation for the tangent plane to the graph of f at the point $(\log(2), \log(2))$ and use it to estimate $f(\log(2)+0.1, \log(2)+0.004)$.

Remark: As usual on the college life as well as in galaxy life, log denotes the natural logarithm. There is no particular reason (except maybe at high school) to think, an other base is natural, except for the fact that we earthlings have 10 fingers. I'm sure you can read this word of wisdom somewhere in a "manual to the galaxy".

Space for work

Problem 7) (10 points)

Consider the graph of the function $h(x, y) = e^{-3x-y} + 4$.

a) (2 points) Find a function $g(x, y, z)$ of three variables such that this surface is the level set of g.
b) (2 points) Find a vector normal to the tangent plane of this surface at (x, y, z).
c) (2 points) Is this tangent plane ever horizontal? Why or why not?
d) (3 points) Give an equation for the tangent plane at $(0, 0)$.

Space for work

Problem 8) (10 points)

Minimize the function $E(x,y,z) = \frac{k^2}{8m}(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2})$ under the constraint $xyz = 8$, where k^2 and m are constants.

Remark. In quantum mechanics, E is the ground state energy of a particle in a box with dimensions x, y, z. The constant k is usually denoted by $\hbar$ and called the Planck constant.

Space for work

Problem 9) (10 points)

Assume $F(x,y) = g(x^2 + y^2)$, where g is a function of one variable. Find $F_{xx}(1,2) + F_{yy}(1,2)$, given that $g'(5) = 3$ and $g''(5) = 7$.

Space for work

Problem 10) (10 points)

Let $g(x,y)$ be the distance to the curve $x^2 + 2y^2 + y^4/10 = 1$.. Show that g is a solution of the partial differential equation

$$f_x^2 + f_y^2 = 1$$

outside the curve.

Hint: here no computations are needed. The shape of the curve is pretty much irrelevant. What does the PDE say about the gradient ∇f?

Remark: This example just needs thought. Use it as a "pillow problem" that is think about it before going to sleep. By the way, the PDE is called **eiconal equation**. It describes wave fronts in optics.

Space for work