COACH TOPIC INTEGRATION Math 21a, Fall 2006

Knowing integrals

In a time of computer algebra systems, knowing integrals is less urgent, but it is imperative to know a few essential antiderivatives:

$\int \sin(x)\, dx = -\cos(x)$	$\int \cos(x)\, dx = \sin(x)$
$\int 1/x\, dx = \log(x)$	$\int \frac{1}{x}\, dx = \log(x)$
$\int 1\, dx = x$	$\int e^x\, dx = e^x$
$\int \sqrt{1+x}\, dx = (1+x)^{3/2}\, \frac{2}{3}$	$\int \frac{1}{1+x^2}\, dx = \arctan(x)$

Here are some integrals which are good to know but which are not so important to have available in memory:

$\int \sinh(x)\, dx = \cosh(x)$	$\int \cosh(x)\, dx = \sinh(x)$
$\int 1/\sqrt{1-x^2}\, dx = \arcsin(x)$	$\int 1/\sqrt{1+x^2}\, dx = \operatorname{arcsinh}(x)$
$\int 1/\sin^2(x)\, dx = -\cot(x)$	$\int 1/\cos^2(x)\, dx = \tan(x)$

Substitution

Integrals of the form $\int f(g(x))g'(x)\, dx$ can be detected on the "spot" as $F(g(x))$, where F is the antiderivative of f. Examples:

$\int \tan(x)\, dx = -\log(\cos(x))$	$\int \cot(x)\, dx = \log(\sin(x))$
$\int 2x\cos(1+x^2)\, dx = \sin(1+x^2)$	$\int x^2\sqrt{1+x^3}\, dx = (1+x^3)^{3/2}\, \frac{2}{9}$
$\int \sin(x)\cos(x)\, dx = -\cos^2(x)/2$	$\int \frac{1}{x\log(x)}\, dx = \log(\log(x))$
$\int \log(x)/x\, dx = \log^2(x)/2$	$\int \sin^5(x)\cos(x)\, dx = \sin^6(x)/6$

Trigonometric forms

Knowing the trigonometric identities $\sin^2(x) + \cos^2(x) = 1, \cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x), \sin(2x) = 2\sin(x)\cos(x)$ is helpful:

$\int \tan^2(x)\, dx = \int \frac{(1-\cos^2(x)}{\cos^2(x)}\, dx$	$\int \cos^2(x)\, dx = \int \frac{1+\cos(2x)}{2}\, dx$
$\int \sin^2(x)\, dx = \int \frac{1-\cos(2x)}{2}\, dx$	$\int \sin^3(x)\, dx = \int \sin(x)(1-\cos^2(x))\, dx$

Integration by parts

Integration often works for products $x^n f(x)$. Integrating the product rule $uv' = uv - u'v$ gives

$$\int u\, dv = uv - \int duv\ .$$

Sometimes, it has to be repeated

$\int x\sin(x)\, dx = -x\cos(x) + \sin(x)$	$\int x\log(x)\, dx = x^2\log(x)/2 - x^2/2$
$\int \log(x)\, dx = \log(x)x - \int \frac{1}{x}x\, dx$	$\int xe^x\, dx = xe^x - \int e^x\, dx$

Sometimes, it has to be repeated:

$\int e^x\cos(x)\, dx = e^x\sin(x) - \int e^x\sin(x) = e^x\sin(x) - e^x\cos(x) - \int e^x\cos(x)\, dx$

$\int x^2e^x\, dx = x^2e^x - \int 2xe^x\, dx = x^2e^x - 2xe^x + \int 2e^x\, dx$

Partial fractions

Rational functions can be integrated by writing the fraction as a sum of simpler fractions:

$\int \frac{1}{1-x^2}dx = \int \frac{A}{1-x}\, dx + \int \frac{B}{1+x}\, dx$

$\int \frac{x}{1+x}dx = \int \frac{1+x}{1+x}\, dx - \int \frac{1}{1+x}\, dx$

Remarks

If you get into a mess with an integral in an exam in this course, it is an indication that you need an other approach:

other coordinate system

change the order of integration

use of an integral theorem (covered later)

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Knowing integrals

$$\int \frac{1}{1+x}\,dx$$

$$\int x^2 + \cos(x) + \log(x)\,dx$$

Substitution

$$\int \sin(x)\cos(x)\,dx$$

$$\int \log^3(x)/x\,dx$$

Trigonometric forms

$$\int \cot^2(x)\,dx$$

$$\int \cos^3(x)\,dx$$

Integration by parts

$$\int x^2 \sin(x)\,dx$$

$$\int (x+1)\sin(x)\,dx$$

Partial fractions

$$\int \frac{1}{(x-1)(x+2)}dx$$

$$\int \frac{1}{(x^2+x)}dx$$