

Name: 

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- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

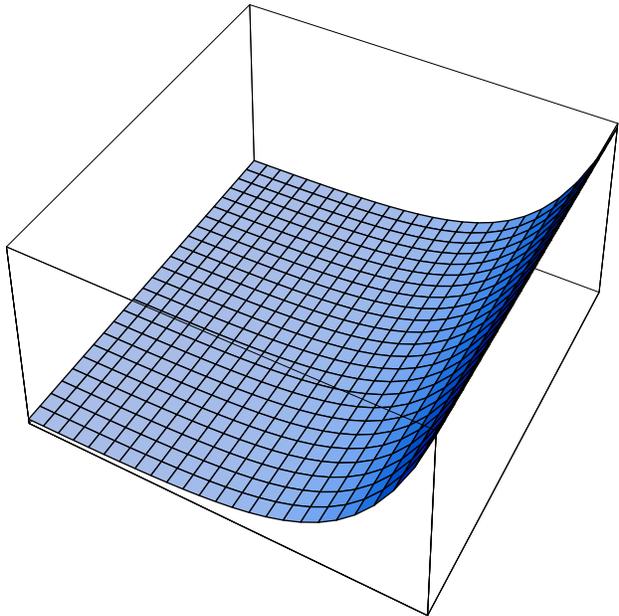
Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

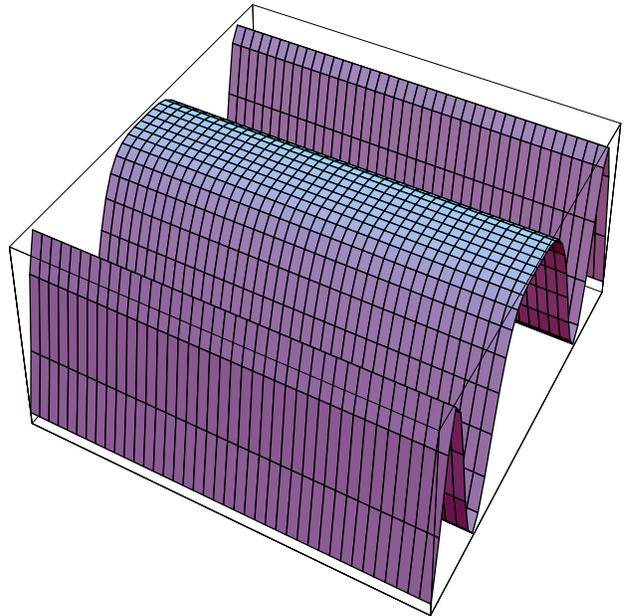
- 1)  T  F The point  $(-1, -1, -1)$  is in spherical coordinate described as  $(\rho, \theta, \phi) = (\sqrt{3}, 5\pi, 3\pi/4)$
- 2)  T  F If  $|\vec{v} \times \vec{w}| = 0$  then  $\vec{v} = \vec{0}$  or  $\vec{w} = \vec{0}$ .
- 3)  T  F The surface  $z^2 + 4y^2 = x^2 + 1$  is a two sheeted hyperboloid.
- 4)  T  F The surface  $4x^2 - 4x + y^2 - 2y - 120 = -z^2$  is an ellipsoid.
- 5)  T  F The parametrized lines  $\vec{u}(t) = \langle 1 + 2t, 2 - 5t, 1 + t \rangle$  and  $\vec{v}(t) = \langle 3 - 4t, -3 + 10t, 2 - 2t \rangle$  are the same line.
- 6)  T  F The surface  $\sin(x) = z$  contains lines which are parallel to the y-axis.
- 7)  T  F If  $\vec{u} \cdot \vec{v} = 0$ ,  $\vec{v} \cdot \vec{w} = 0$  and  $\vec{v}$  is not the zero vector, then  $\vec{u} \cdot \vec{w} = 0$ .
- 8)  T  F The curvature of a curve depends upon the speed at which one travels upon it.
- 9)  T  F Two lines in space that do not intersect must be parallel.
- 10)  T  F A line in space can intersect an elliptic paraboloid in 4 points.
- 11)  T  F If  $\vec{u} \times \vec{v} = 0$  and  $\vec{u} \cdot \vec{v} = 0$ , then one of the vectors  $\vec{u}$  and  $\vec{v}$  is zero.
- 12)  T  F If the velocity vector  $\vec{r}'(t)$  and the acceleration vector  $\vec{r}''(t)$  of a curve are parallel at time  $t = 1$ , then the curvature  $\kappa(t)$  of the curve is zero at time  $t = 1$ .
- 13)  T  F If the speed of a parametrized curve is constant over time, then the curvature of the curve  $\vec{r}(t)$  is zero.
- 14)  T  F The length of the vector projection of a vector  $\vec{v}$  onto a vector  $\vec{w}$  is always equal to the length of the vector projection of  $\vec{w}$  onto  $\vec{v}$ .
- 15)  T  F A quadric  $ax^2 + by^2 + cz^2 = 1$  is contained in the interior of a sphere  $x^2 + y^2 + z^2 < 100$ , then the constants  $a, b, c$  are all positive and the quadric is an ellipsoid.
- 16)  T  F There is a hyperboloid of the form  $ax^2 + by^2 - cz^2 = 1$  which has a trace which is a parabola.
- 17)  T  F The set of points in space which have distance 1 from the line  $x = y = z$  form a cylinder.
- 18)  T  F The velocity vector of a parametric curve  $\vec{r}(t)$  always has constant length.
- 19)  T  F The volume of a parallelepiped spanned by  $\vec{u}, \vec{v}, \vec{w}$  is  $|(\vec{u} \times \vec{v}) \cdot \vec{w}|$ .
- 20)  T  F The equation  $x^2 + y^2/4 = 1$  in space describes an ellipsoid.

Problem 2a) (3 points)

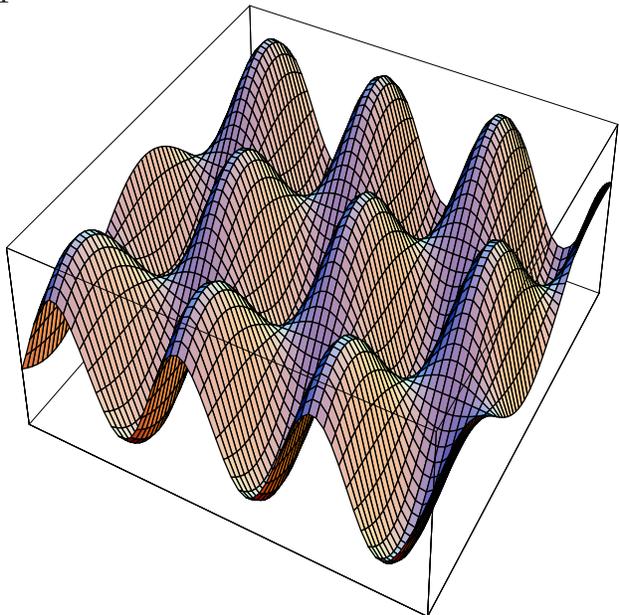
Match the equation with their graphs. No justifications are needed.



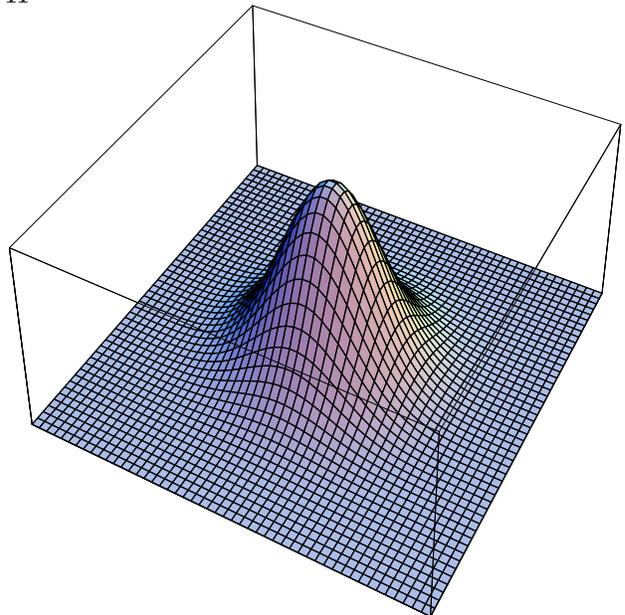
I



II



III

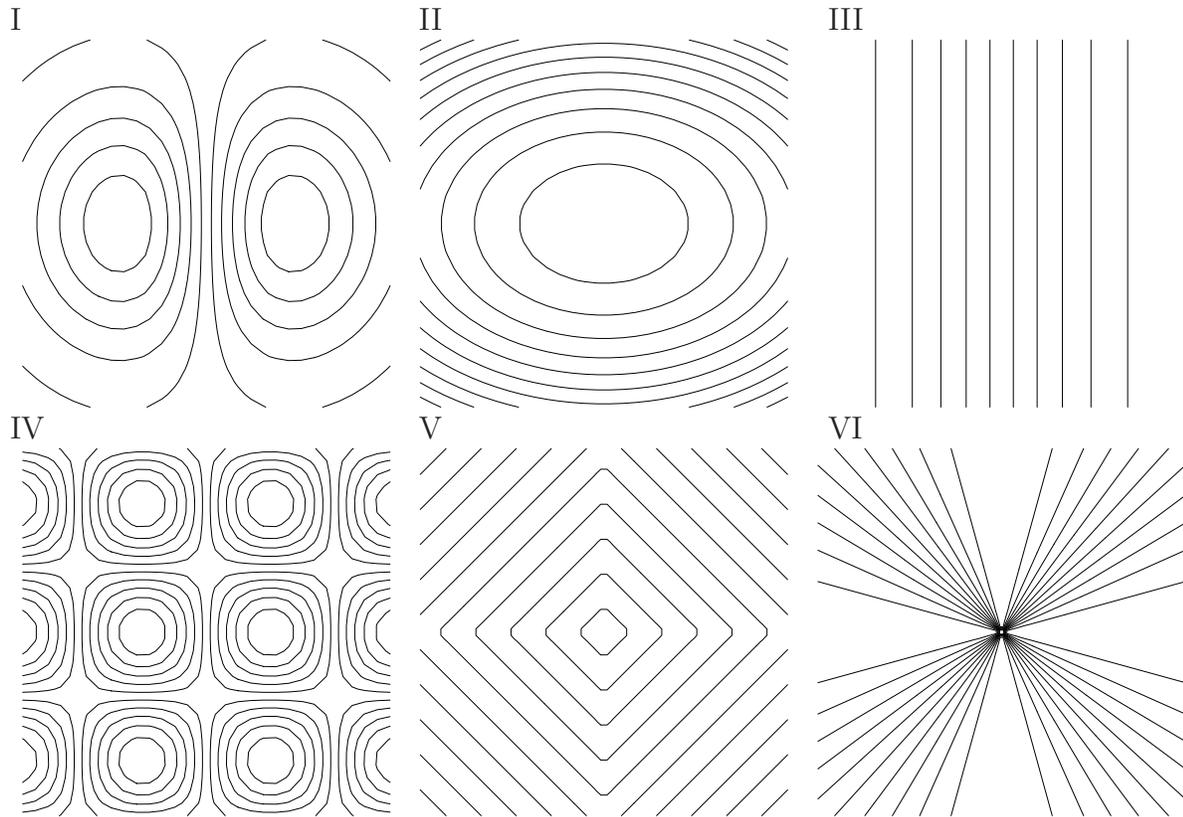


IV

Enter I,II,III,IV here	Equation
	$z = \sin(5x) \cos(2y)$
	$z = \cos(y^2)$
	$z = e^{-x^2-y^2}$
	$z = e^x$

Problem 2b) (4 points)

Match the contour maps with the corresponding functions  $f(x, y)$  of two variables. No justifications are needed.



Enter I,II,III,IV,V or VI here	Function $f(x, y)$
	$f(x, y) = \sin(x)$
	$f(x, y) = x^2 + 2y^2$
	$f(x, y) =  x  +  y $
	$f(x, y) = \sin(x) \cos(y)$
	$f(x, y) = xe^{-x^2-y^2}$
	$f(x, y) = x^2/(x^2 + y^2)$

Match the following points in cartesian coordinates with the points in spherical coordinates:

a)  $(x, y, z) = (\sqrt{2}, 0, 0)$

b)  $(x, y, z) = (0, \sqrt{2}, 0)$

c)  $(x, y, z) = (0, 0, \sqrt{2})$

d)  $(x, y, z) = (1, 1, 0)$

e)  $(x, y, z) = (1, 0, 1)$

f)  $(x, y, z) = (0, 1, 1)$

1)  $(\rho, \phi, \theta) = (\sqrt{2}, 0, 0)$ .

2)  $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, \pi/4)$ .

3)  $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, 0)$ .

4)  $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, \pi/2)$ .

5)  $(\rho, \phi, \theta) = (\sqrt{2}, \pi/4, \pi/2)$ .

6)  $(\rho, \phi, \theta) = (\sqrt{2}, \pi/4, 0)$ .

Problem 3) (10 points)

- a) (7 points) Find a parametric equation for the line which is the intersection of the two planes  $2x - y + 3z = 9$  and  $x + 2y + 3z = -7$ .
- b) (3 points) Find a plane perpendicular to both planes and which passes through the point  $P = (1, 1, 1)$ .

Problem 4) (10 points)

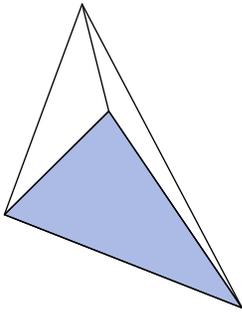
Given the vectors  $\vec{v} = \langle 1, 1, 0 \rangle$  and  $\vec{w} = \langle 0, 0, 1 \rangle$  and the point  $P = (2, 4, -2)$ . Let  $\Sigma$  be the plane which goes through the origin  $(0, 0, 0)$  and which contains the vectors  $\vec{v}$  and  $\vec{w}$ . Let  $S$  be the unit sphere  $x^2 + y^2 + z^2 = 1$ .

- a) (6 points) Compute the distance from  $P$  to the plane  $\Sigma$ .
- b) (4 points) Find the shortest distance from  $P$  to the sphere  $S$ .

Problem 5) (10 points)

- a) (6 points) Find an equation for the plane through the points  $A = (0, 1, 0)$ ,  $B = (1, 2, 1)$  and  $C = (2, 4, 5)$ .
- b) (4 points) Given an additional point  $P = (-1, 2, 3)$ , what is the volume of the tetrahedron which has  $A, B, C, P$  among its vertices.

**A useful fact which you can use without justification in b):** the volume of the tetrahedron is  $1/6$  of the volume of the parallelepiped which has  $AB, AC,$  and  $AP$  among its edges.



Problem 6) (10 points)

The parametrized curve  $\vec{u}(t) = \langle t, t^2, t^3 \rangle$  (known as the "twisted cubic") intersects the parametrized line  $\vec{v}(s) = \langle 1 + 3s, 1 - s, 1 + 2s \rangle$  at a point  $P$ . Find the angle of intersection.

Problem 7) (10 points)

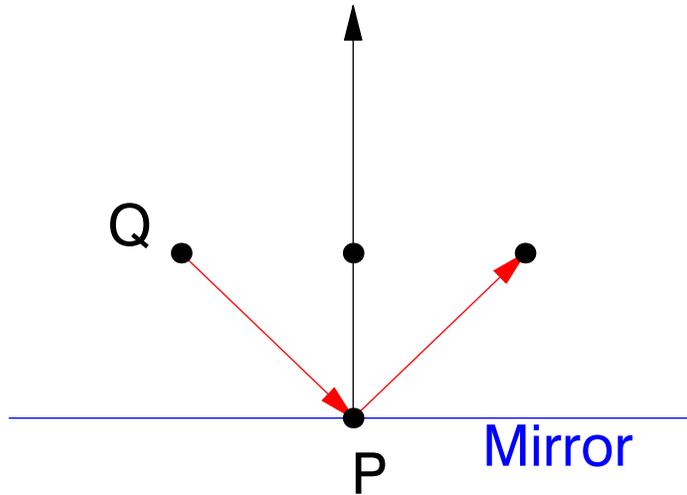
Let  $\vec{r}(t)$  be the space curve  $\vec{r}(t) = (\log(t), 2t, t^2)$ , where  $\log(t)$  is the natural logarithm (denoted by  $\ln(t)$  in some textbooks).

- What is the velocity and what is the acceleration at time  $t = 1$ ?
- Find the length of the curve from  $t = 1$  to  $t = 2$ .

Problem 8) (10 points)

A planar mirror in space contains the point  $P = (4, 1, 5)$  and is perpendicular to the vector  $\vec{n} = \langle 1, 2, -3 \rangle$ . The light ray  $\vec{QP} = \vec{v} = \langle -3, 1, -2 \rangle$  with source  $Q = (7, 0, 7)$  hits the mirror plane at the point  $P$ .

- (4 points) Compute the projection  $\vec{u} = \vec{P}_{\vec{n}}(\vec{v})$  of  $\vec{v}$  onto  $\vec{n}$ .
- (6 points) Identify  $\vec{u}$  in the figure and use it to find a vector parallel to the reflected ray.



Problem 9) (10 points)

We know the acceleration  $\vec{r}''(t) = \langle 2, 1, 3 \rangle + t\langle 1, -1, 1 \rangle$  and the initial position  $\vec{r}(0) = \langle 0, 0, 0 \rangle$  and initial velocity  $\vec{r}'(0) = \langle 11, 7, 0 \rangle$  of an unknown curve  $\vec{r}(t)$ . Find  $\vec{r}(6)$ .

Problem 10) (10 points)

Intersecting the elliptic cylinder  $x^2 + y^2/4 = 1$  with the plane  $z = \sqrt{3}x$  gives a curve in space.

- (3 points) Find the parametrization of the curve.
- (3 points) Compute the unit tangent vector  $\vec{T}$  to the curve at the point  $(0, 2, 0)$ .
- (4 points) Write down the arc length integral and evaluate the arc length of the curve.