Name:

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MWF 10 Andrew Cotton-Clay
MWF 10 Oliver Knill
MWF 10 HT Yau
MWF 11 Ana Caraiani
MWF 11 Chris Phillips
MWF 11 Ethan Street
MWF 12 Toby Gee
MWF 12 Xinwen Zhu
TTH 10 Jack Huizenga
TTH 10 Fred van der Wyck
TTH 11:30 Ming-Tao Chuan
TTH 11:30 Fred van der Wyck

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-2, we need to see details of your computation.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F The surface described in spherical coordinates as $\phi = \pi/4$ is the xy plane.

2) T F The length of the unit tangent vector $\vec{T}$ for a curve $\vec{r}(t)$ is independent of t.

3) T F For all vectors $\vec{v}$ and $\vec{w}$ the vector $\vec{w} \times (\vec{w} \times \vec{v})$ is perpendicular to $\vec{v}$.

4) T F There is a point (x, y, z) in space, for which the cylindrical coordinates (r, θ, z) and spherical coorindates (ρ, θ, ϕ) satisfy $(r, \theta, z) = (\rho, \theta, \phi - \pi/2)$.

5) T F The two planes $x + y - z = 1$ and $-x - y + z = 2$ intersect in a line.

6) T F $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = 0$ implies $|u| = |v|$.

7) T F The contour curves $\sin(x) + y = 1$ and $\sin(x) + y = 2$ do not intersect.

8) T F There is a vector $\vec{v}$ for which the vector projection $\text{proj}_{\vec{v}}(\vec{j})$ is equal to $2\vec{j}$.

9) T F $(\vec{k} \times \vec{i}) \times \vec{i} = \vec{j} \times (\vec{i} \times \vec{k})$

10) T F If a curve $\vec{r}(t)$ lies in a plane, goes through the point $(1, 1, 1)$, and has the binormal vector $\vec{B}(t) = \langle 3, 4, 5 \rangle$, then the plane is $3x + 4y + 5z = 12$.

11) T F The angle between $\vec{r}\,'(t)$ and $\vec{r}\,''(t)$ is always 90 degrees.

12) T F A line intersects a hyperbolic paraboloid always in 2 distinct points.

13) T F There is a quadric surface, each of whose intersections with the coordinate planes is either an ellipse or a parabola.

14) T F The equation $x^2 - y^2 - z^2 = 1$ defines a one-sheeted hyperboloid.

15) T F The function $f(x, y) = 1/(1 + x^2 + y^2)$ is continuous everywhere.

16) T F If the number $\vec{u} \cdot (\vec{v} \times \vec{w})$ is positive, then $(\vec{w} \times \vec{v}) \cdot \vec{u}$ is positive.

17) T F The number $|\vec{u} \times (\vec{v} \times \vec{w})|$ is the volume of the parallelepiped spanned by $\vec{u}, \vec{v}$ and $\vec{w}$.

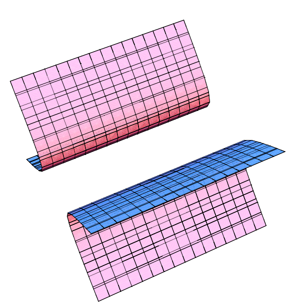
18) T F The set of points P for which the distance of P to the point $(0, 0, 0)$ is 1 less than the distance to the point $(0, 0, 2)$ is a paraboloid.

19) T F If $\vec{v}, \vec{w}$ are two nonzero vectors, then the projection vector $\text{proj}_{\vec{w}}(\vec{v})$ can be longer than $\vec{v}$.

20) T F The number $|\vec{v} \times \vec{w}|$ is the area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$.

Problem 2a) (5 points)

Match the equations with the pictures. No justifications are necessary in this problem.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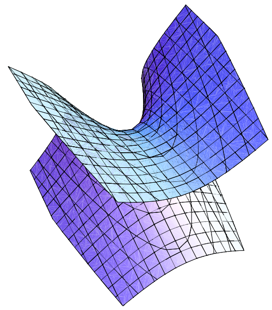
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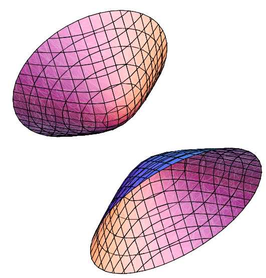
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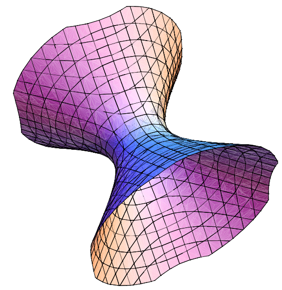
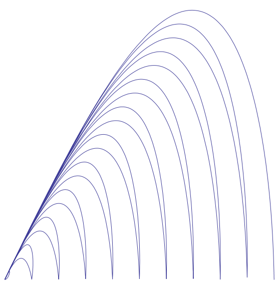

IV

Enter I,II,III,IV here	Equation
	$x^2 + y - z^2 - 1 = 0$
	$y^2 - 2z^2 - 1 = 0$
	$x^2 - y^2 + z^2 + 1 = 0$
	$x^2 - y^2 + z^2 - 1 = 0$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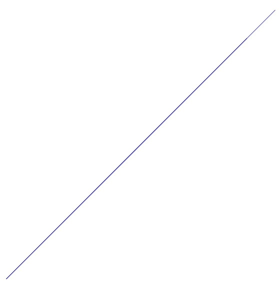
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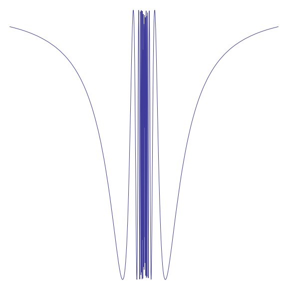
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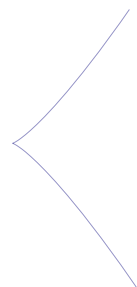
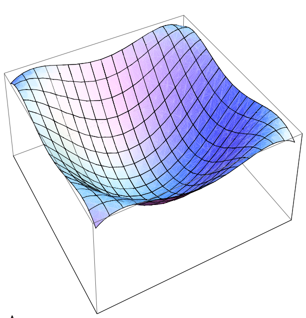

4

Enter 1,2,3,4 here	Equation
	$\langle t^4, 1 + t^5 \rangle$
	$\langle t\cos(5t), t\cos(5t) \rangle$
	$\langle \lvert t\cos(5t)\rvert, \lvert t\sin(10t)\rvert \rangle$
	$\langle 3 + 2t, \cos(1/t) \rangle$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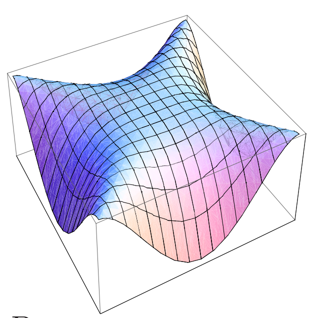
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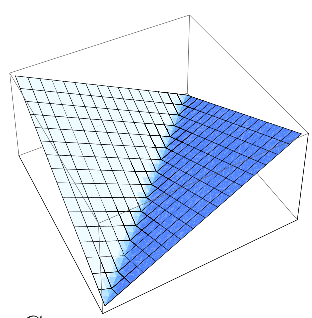
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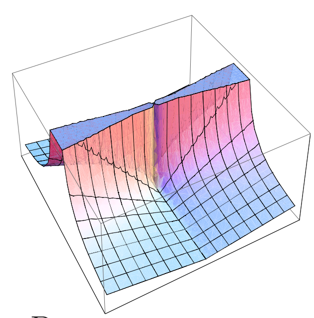
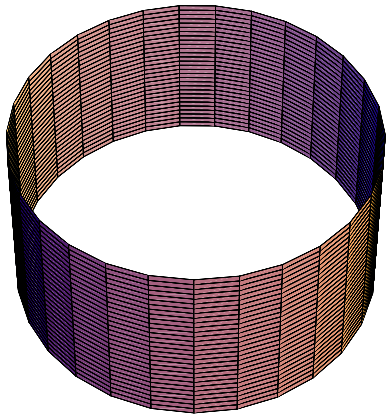

D

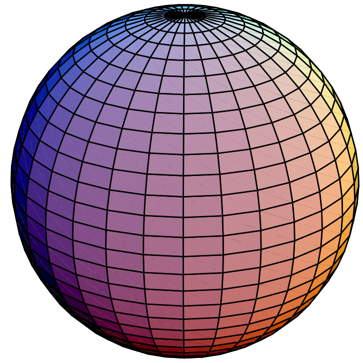
Enter A,B,C,D here	Equation
	$f(x, y) = \lvert x/y \rvert$
	$f(x, y) = \sin(x^2 + y^2)$
	$f(x, y) = \lvert x - y \rvert$
	$f(x, y) = \cos(x^2 - y^2)$

Problem 2b) (5 points)

Match the surfaces with their parametrizations as well as with the description either in cylindrical coordinates (r, θ, z) or in spherical coordinates (ρ, ϕ, θ).



I



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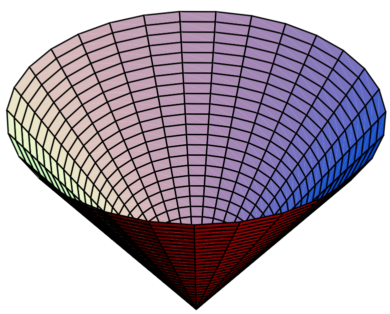

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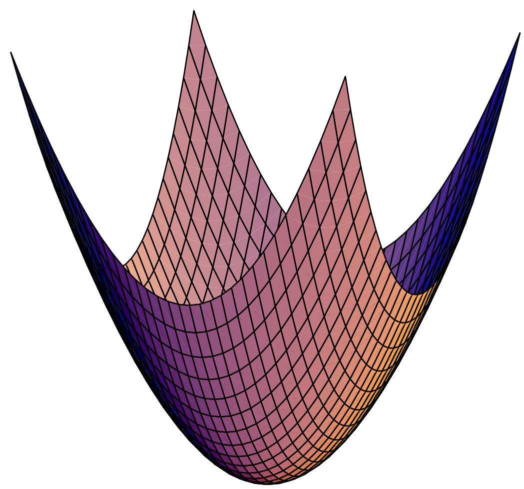


IV

Enter I,II,III,IV here	Parametrization of the surface
	$\langle 3\cos(\theta), 3\sin(\theta), 2z \rangle$.
	$\langle x, y, 3x^2 + 3y^2 \rangle$
	$\langle 3\sin(\phi)\cos(\theta), 3\sin(\phi)\sin(\theta), 3\cos(\phi) \rangle$
	$\langle 3z\cos(\theta), 3z\sin(\theta), 2z \rangle$

Enter I,II,III,IV here	Description in cylindrical or spherical coordinates
	$z = 3r^2$
	$3z = 2r$
	$\rho = 3$
	$r = 3$

Problem 3) (10 points)

A tetrahedron has the vertices $A = (1,1,0), B = (3,2,0), C = (2,1,1), D = (3,2,1)$ with base triangle A, B, C.

a) (5 points) Find the height of the tetrahedron.

b) (5 points) The volume of a tetrahedron is the base area times height divided by 3. What is the volume of the tetrahedron with vertices A,B,C,D.

Problem 4) (10 points)

Find the symmetric equation

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

of the intersection of the two planes

$$2x + y + z = 4$$

and

$$x - y + 2z = 5 .$$

Problem 5) (10 points)

What is the distance between the two cylinders $x^2 + y^2 = 1$ and $(z - 2)^2 + (x - 5)^2 = 4$?

Problem 6) (10 points)

Find the arc length of the parameterized curve

$$\vec{r}(t) = \langle 2\sin(t), \frac{t^4}{4} + \frac{1}{2t^2}, 2\cos(t) \rangle$$

from $t = 1$ to $t = 2$.

Problem 7) (10 points)

At time $t = 0$ two trapeze artists have positions $\vec{r}(0) = \langle 0, 0, 25 \rangle$ and $\vec{s}(0) = \langle 10, 0, 23 \rangle$ and velocities $\vec{r}\,'(0) = \langle 2, 0, 1 \rangle$ and $\vec{s}\,'(0) = \langle -3, 0, 2 \rangle$. They both experience a constant gravitational acceleration $\langle 0, 0, -10 \rangle$. Find the paths $\vec{r}(t), \vec{s}(t)$ and determine at which point the artists meet.

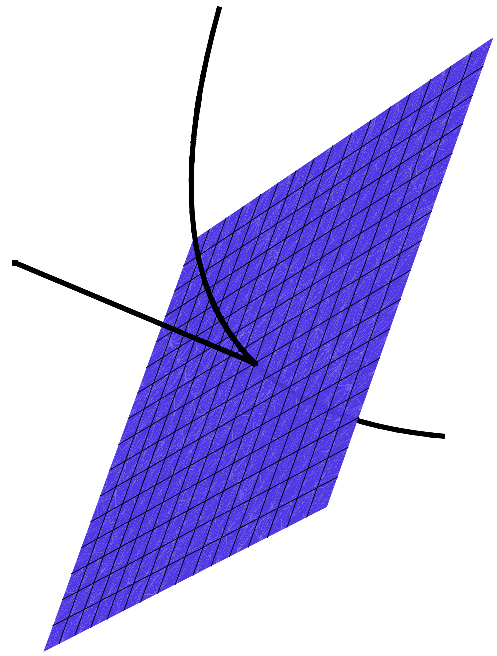
Problem 8) (10 points)

The angle between a curve and a plane is defined as $\pi/2 - \alpha$, where α is the angle between the normal vector to the plane and the velocity vector of the curve at the point of intersection.

a) (3 points) Find a normal vector to the plane $x + y - z/2 = 1$.

b) (3 points) What is the velocity vector to the curve $C : \vec{r}(t) = \langle 1, 0, 0 \rangle + t\langle 1, 1, 3 \rangle + t^2\langle 1, 1, 1 \rangle$ at time $t = 0$?

c) (4 points) Find the angle (in radians) between the plane $x + y - z/2 = 1$ and the curve C at the point of intersection $\vec{r}(0)$.



Problem 9) (10 points)

The intersection of the paraboloid

$$x^2 + y^2 - z = 5$$

with the plane

$$x + y = 5$$

is a curve. Find the parametrization of this curve.

Problem 10) (10 points)

a) (3 points) Parametrize the plane containing the three points $A = (1, 1, 1), B = (1, 3, 2)$ and $C = (3, 4, 5)$.
b) (4 points) Parametrize the sphere which is centered at $(1, 1, 1)$ and has radius 3.
c) (3 points) Parametrize the surface which is given in spherical coordinates as $\rho = 3 + \sin(\phi)\sin(\theta)$.