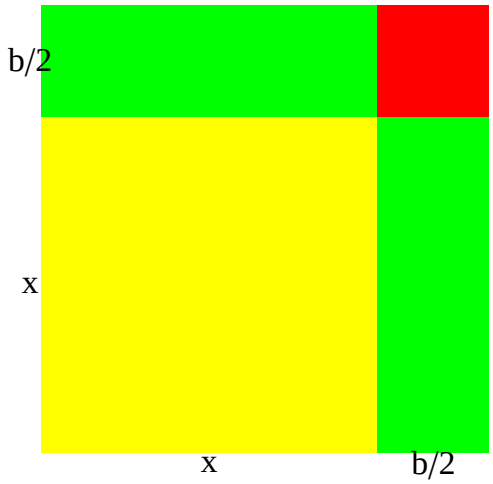
In-Class-Exercice (ICE) **Math21a, Fall 2009, Oliver Knill**

Completion of the square.

Reminder: The quadratic equation $x^2 + bx + c = 0$ is solved by adding $(b/2)^2$ on both sides:

$$\begin{aligned} x^2 + bx &= -c \\ x^2 + bx + (b/2)^2 &= (b/2)^2 - c \\ (x + b/2)^2 &= (b/2)^2 - c \\ x + b/2 &= \sqrt{(b/2)^2 - c} \\ x &= \pm\sqrt{(b/2)^2 - c} - b/2 \ . \end{aligned}$$



The picture shows why this is called the "completion of the square". Adding a square of area $b^2/4$ completes the large square.

The method can be used to find the center of a circle or sphere:

1. Find the center and radius of the circle
$$2x^2 + 4x + 2y^2 = 16$$

2. Find the center and radius of the sphere
$$x^2 + 6x + y^2 - 4y + z^2 = 4 + 2z$$

3. Do the spheres
$$x^2 - 4x + y^2 + 2y + z^2 = 3, x^2 + y^2 - 2y + z^2 - 10z = -12$$
intersect?