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- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work.** Except for problems 1-2 and 8, we need to see details of your computation.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

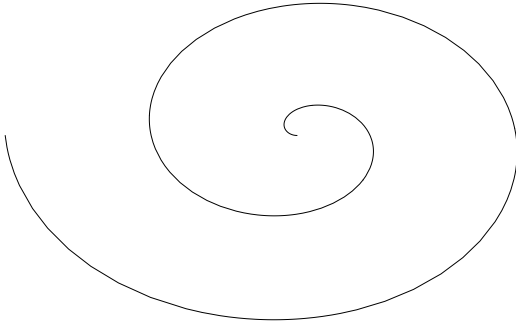
Problem 1) TF questions (20 points) No justifications needed

- 1)  T  F The length of the sum of two vectors is always the sum of the length of the vectors.
- 2)  T  F For any three vectors,  $\vec{v} \times (\vec{w} + \vec{u}) = \vec{w} \times \vec{v} + \vec{u} \times \vec{v}$ .
- 3)  T  F The set of points which satisfy  $x^2 + 2x + y^2 - z^2 = 0$  is a cone.
- 4)  T  F The surface  $\vec{r}(u, v) = \langle \cos(u^2) \sin(v^2), \sin(u^2) \sin(v^2), \cos(v^2) \rangle$  with  $0 \leq u < \sqrt{2\pi}, 0 \leq v \leq \sqrt{\pi}$  is a sphere.
- 5)  T  F If  $P, Q, R$  are 3 different points in space that don't lie in a line, then  $\vec{PQ} \times \vec{RQ}$  is a vector orthogonal to the plane containing  $P, Q, R$ .
- 6)  T  F The line  $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$  hits the plane  $2x + 3y + 4z = 9$  at a right angle.
- 7)  T  F The function  $f(x, y) = \sin(xy)/y$  is continuous everywhere.
- 8)  T  F For any two vectors,  $\vec{v} \times \vec{w} = \vec{w} \times \vec{v}$ .
- 9)  T  F If  $|\vec{v} \times \vec{w}| = 0$  for all vectors  $\vec{w}$ , then  $\vec{v} = \vec{0}$ .
- 10)  T  F If  $\vec{u}$  and  $\vec{v}$  are orthogonal vectors, then  $(\vec{u} \times \vec{v}) \times \vec{u}$  is parallel to  $\vec{v}$ .
- 11)  T  F Every vector contained in the plane  $x + y + z = 1$  is parallel to the vector  $\langle 1, 1, 1 \rangle$ .
- 12)  T  F The sphere can in cylindrical coordinates described as  $r^2 = 1 - z^2$ .
- 13)  T  F The curvature of the curve  $2\vec{r}(4t)$  at  $t = 0$  is twice the curvature of the curve  $\vec{r}(t)$  at  $t = 0$ .
- 14)  T  F The set of points which satisfy  $x^2 - 2y^2 - 3z^2 = 0$  form an ellipsoid.
- 15)  T  F If  $\vec{v} \times \vec{w} = (0, 0, 0)$ , then  $\vec{v} = \vec{w}$ .
- 16)  T  F Every vector contained in the line  $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$  is parallel to the vector  $\langle 1, 1, 1 \rangle$ .
- 17)  T  F Two nonzero vectors are parallel if and only if their cross product is  $\vec{0}$ .
- 18)  T  F The vector  $\vec{u} \times (\vec{v} \times \vec{w})$  is always in the same plane together with  $\vec{v}$  and  $\vec{w}$ .
- 19)  T  F The line  $\vec{r}(t) = \langle 1 + 2t, 1 + 2t, 1 - 4t \rangle$  hits the plane  $x + y + z = 9$  at a right angle.
- 20)  T  F The intersection of the ellipsoid  $x^2/3 + y^2/4 + z^2/3 = 1$  with the plane  $y = 1$  is a circle.

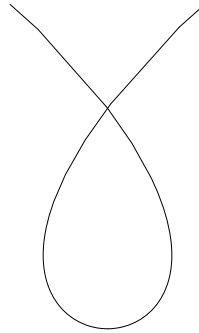
Problem 2a) (3 points)

Match the curves with their parametric definitions.

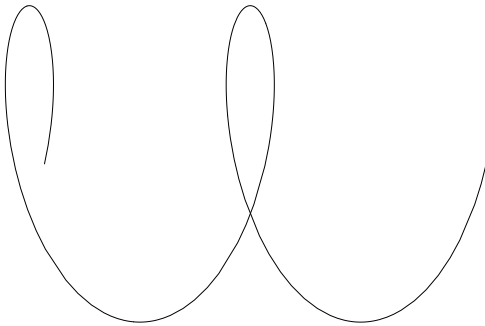
I



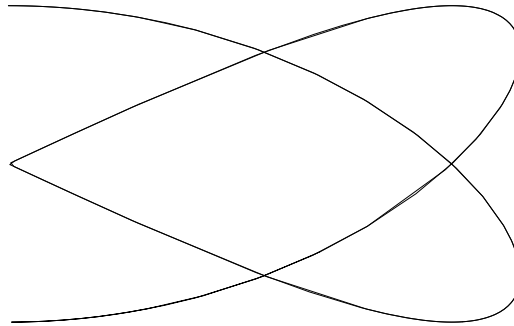
II



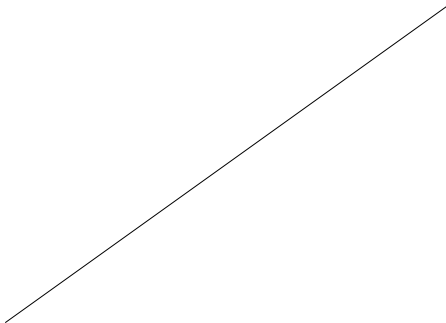
III



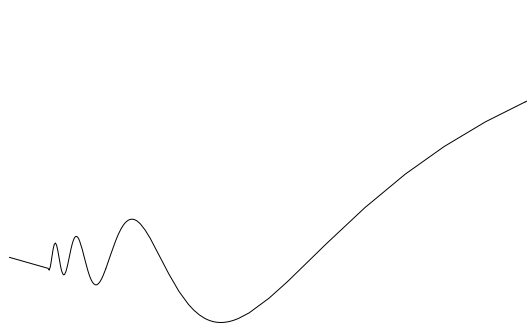
IV



V



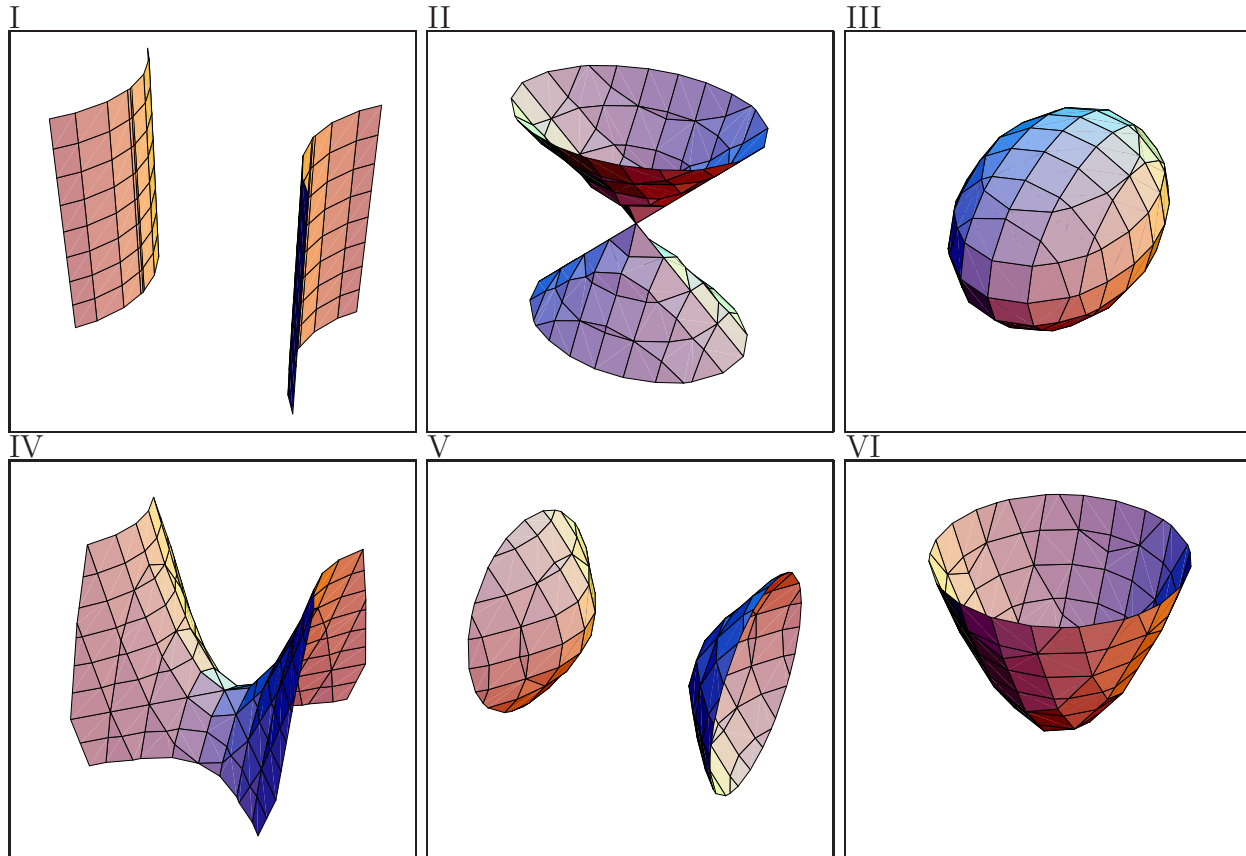
VI



Enter I,II,III,IV,V or VI here	Parametric equation for the curve
	$\vec{r}(t) = \langle t, \sin(1/t)t \rangle$
	$\vec{r}(t) = \langle t^3 - t, t^2 \rangle$
	$\vec{r}(t) = \langle t + \cos(2t), \sin(2t) \rangle$
	$\vec{r}(t) = \langle  \sin(2t) , \cos(3t) \rangle$
	$\vec{r}(t) = \langle 1 + t, 5 + 3t \rangle$
	$\vec{r}(t) = \langle -t \cos(t), 2t \sin(t) \rangle$

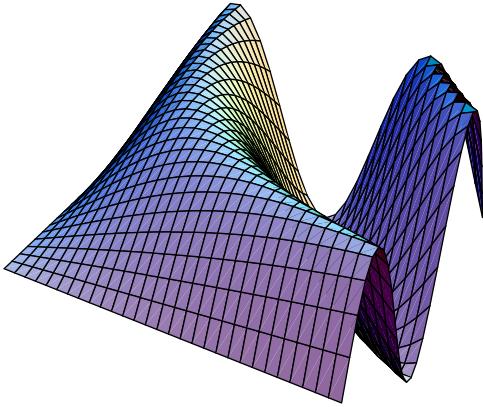
Problem 2b) (3 points)

Match the equations with the surfaces.

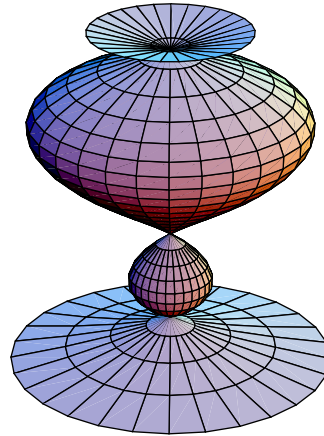


Enter I,II,III,IV,V,VI here	Equation
	$x^2 - y^2 - z^2 = 1$
	$x^2 + 2y^2 = z^2$
	$2x^2 + y^2 + 2z^2 = 1$
	$x^2 - y^2 = 5$
	$x^2 - y^2 - z = 1$
	$x^2 + y^2 - z = 1$

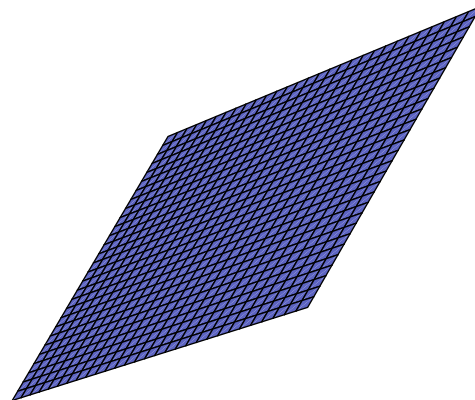
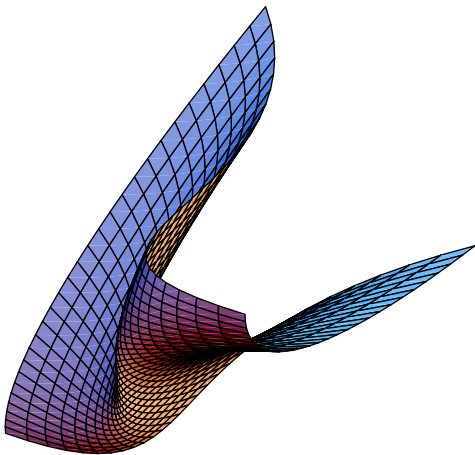
Match the parametric surfaces with their parameterization. No justification is needed.



I



II



III

Enter I,II,III,IV here	Parameterization
	$\vec{r}(u, v) = \langle u, v, u + v \rangle$
	$\vec{r}(u, v) = \langle u, v, \sin(uv) \rangle$
	$\vec{r}(u, v) = \langle 0.2 + u(1 - u^2) \cos(v), (0.2 + u(1 - u^2)) \sin(v), u \rangle$
	$\vec{r}(u, v) = \langle u^3, (u - v)^2, v \rangle$

Problem 3) (10 points)

a) (6 points) Find a parameterization of the line of intersection of the planes  $3x - 2y + z = 7$  and  $x + 2y + 3z = -3$ .

b) (4 points) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing that line.

Problem 4) (10 points)

a) (4 points) Find the area of the parallelogram with vertices  $P = (1, 0, 0)$ ,  $Q = (0, 2, 0)$ ,  $R = (0, 0, 3)$  and  $S = (-1, 2, 3)$ .

b) (3 points) Verify that the triple scalar product has the property  $[\vec{u} + \vec{v}, \vec{v} + \vec{w}, \vec{w} + \vec{u}] = 2[\vec{u}, \vec{v}, \vec{w}]$ .

c) (3 points) Verify that the triple scalar product  $[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$  has the property

$$|[\vec{u}, \vec{v}, \vec{w}]| \leq \|\vec{u}\| \cdot \|\vec{v}\| \cdot \|\vec{w}\|$$

Problem 5) (10 points)

Find the distance between the two lines

$$\vec{r}_1(t) = \langle t, 2t, -t \rangle$$

and

$$\vec{r}_2(t) = \langle 1 + t, t, t \rangle .$$

Problem 6) (10 points)

Find an equation for the plane that passes through the origin and whose normal vector is parallel to the line of intersection of the planes  $2x + y + z = 4$  and  $x + 3y + z = 2$ .

Problem 7) (10 points)

The intersection of the two surfaces  $x^2 + \frac{y^2}{2} = 1$  and  $z^2 + \frac{y^2}{2} = 1$  consists of two curves.

- a) (4 points) Parameterize each curve in the form  $\vec{r}(t) = (x(t), y(t), z(t))$ .
- b) (3 points) Set up the integral for the arc length of one of the curves.
- c) (3 points) What is the arc length of this curve?

Problem 8) (10 points)

- a) (6 points) Find the curvature  $\kappa(t)$  of the space curve  $\vec{r}(t) = \langle -\cos(t), \sin(t), -2t \rangle$  at the point  $\vec{r}(0)$ .
- b) (4 points) Find the curvature  $\kappa(t)$  of the space curve  $\vec{r}(t) = \langle -\cos(5t), \sin(5t), -10t \rangle$  at the point  $\vec{r}(0)$ .

**Hint.** Use one of the two formulas for the curvature

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3},$$

where  $\vec{T}(t) = \vec{r}'(t)/|\vec{r}'(t)|$ . The curvatures in b) can be derived from the curvature in a). There is no need to redo the calculation in b) if you give a proper justification.

Problem 9) (10 points)

For each of the following, fill in the blank with  $<$  (less than),  $>$  (greater than), or  $=$  (equal).

Justify your answer completely.

1. The arc length of the curve parameterized by  $\vec{f}(t) = \langle \cos 2t, 0, \sin 2t \rangle$ ,  $0 \leq t \leq \pi$ .

The arc length of the curve parameterized by  $\vec{g}(u) = \langle 3, 2 \cos u^2, 2 \sin u^2 \rangle$ ,  $0 \leq u \leq \sqrt{\pi}$ .

2. The arc length of the curve parameterized by  $\vec{f}(t) = \langle t^2, 2 \cos t, 2 \sin t \rangle$ ,  $0 \leq t \leq 2\pi$ .

The arc length of the curve parameterized by  $\vec{g}(u) = \langle u^4, 2 \cos u^2, 2 \sin u^2 \rangle$ ,  $0 \leq u \leq 2\pi$ .

3. The arc length of the curve parameterized by  $\vec{f}(t) = \langle 1 + 3t^2, 2 - t^2, 5 + 2t^2 \rangle$ ,  $0 \leq t \leq 1$ .

The arc length of the curve parameterized by  $\vec{g}(u) = \langle \frac{1}{2}u^2, u, \frac{2\sqrt{2}}{3}u^{3/2} \rangle$ ,  $0 \leq u \leq 2$ .

4. The arc length of the curve parameterized by  $\vec{f}(t) = \langle \sin t, \cos t, t \rangle$ ,  $1 \leq t \leq 5$ .

The arc length of the curve parameterized by  $\vec{g}(u) = \langle u \sin u, u \cos u, u \rangle$ ,  $1 \leq u \leq 5$ .

Problem 10) (10 points)
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Given the plane  $x + y + z = 6$  containing the point  $P = (2, 2, 2)$ . Given is also a second point  $Q = (3, -2, 2)$ .

a) (5 points) Find the equation  $ax + by + cz = d$  for the plane through  $P$  and  $Q$  which is perpendicular to the plane  $x + y + z = 6$ .

b) (5 points) Find the symmetric equation for the intersection of these two planes.



