Name:

MWF 9 Chao Li
MWF 9 Thanos Papaïoannou
MWF 10 Emily Riehl
MWF 10 Jameel Al-Aidroos
MWF 11 Oliver Knill
MWF 11 Tatyana Kobylyatskaya
MWF 12 Tatyana Kobylyatskaya
MWF 12 Yu-Jong Tzeng
TTH 10 Junecue Suh
TTH 10 Pei-Yu Tsai
TTH 11:30 Paul Bourgade

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see details of your computation.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
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9		10
10		10
11		10
12		10
13		10
14		10
Total:		150

Problem 1) True/False questions (20 points)

1)	T	F	For any two vectors $\vec{v}$ and $\vec{w}$ one has $\text{proj}_{\vec{v}}(\vec{v} \times \vec{w}) = \vec{0}$.
2)	T	F	Any parameterized surface S is either the graph of a function $f(x, y)$ or a surface of revolution.
3)	T	F	If the directional derivative $D_{\vec{v}}(f)$ of f into the direction of a unit vector $\vec{v}$ is zero, then $\vec{v}$ is perpendicular to the level curve of f.
4)	T	F	The linearization $L(x, y)$ of $f(x, y) = 5x - 100y$ at $(0, 0)$ satisfies $L(x, y) = 5x - 100y$.
5)	T	F	If a parameterized curve $\vec{r}(t)$ intersects a surface $\{f = c\}$ at a right angle, then at the point of intersection we have $\nabla f(\vec{r}(t)) \times \vec{r}\,'(t) = 0$.
6)	T	F	The curvature of the curve $\vec{r}(t) = \langle \cos(3t^2), \sin(6t^2) \rangle$ at the point $\vec{r}(1)$ is larger than the curvature of the curve $\vec{r}(t) = \langle 2\cos(3t), 2\sin(6t) \rangle$ at the point $\vec{r}(1)$.
7)	T	F	At every point (x, y, z) on the hyperboloid $4x^2 - y^2 + z^2 = 10$, the vector $\langle 4x, -y, z \rangle$ is parallel to the hyperboloid.
8)	T	F	The set $\{\phi = \pi/2, \theta = \pi/2\}$ in spherical coordinates is the positive y-axis.
9)	T	F	The integral $\int_0^1 \int_0^{2\pi} r^2 \sin(\theta) \; d\theta \, dr$ is equal to the area of the unit disk.
10)	T	F	If three vectors $\vec{u}, \vec{v}$ and $\vec{w}$ attached at the origin are in a common plane, then $\vec{u} \cdot ((\vec{v} + \vec{u}) \times \vec{w}) = 0$.
11)	T	F	If a function $f(x, y)$ has a local minimum at $(0, 0)$, then the discriminant D must be positive.
12)	T	F	The integral $\int_0^1 \int_y^1 f(x, y) \, dy \, dx$ represents a double integral over a bounded region in the plane.
13)	T	F	The following identity is true: $\int_0^3 \int_0^x x^2 \, dydx = \int_0^3 \int_y^3 x^2 \, dx \, dy$
14)	T	F	There is a quadric of the form $ax^2 + by^2 + cz^2 = d$, for which all three traces are hyperbola.
15)	T	F	The curvature of a space curve $\vec{r}(t)$ is a vector perpendicular to the acceleration vector $\vec{r}\,''(t)$.
16)	T	F	Assume S is the unit sphere oriented so that the normal vector points outside. Let S^+ be the upper hemisphere and S^- the lower hemisphere. If a vector field $\vec{F}$ has divergence zero, then the flux of $\vec{F}$ through S^+ is equal to the flux of $\vec{F}$ through S^-.
17)	T	F	If a vector field $\vec{F}$ is defined at all points in three-space except at the origin and $\text{curl}(\vec{F}) = \vec{0}$ everywhere, then the line integral of $\vec{F}$ around any closed path not passing through the origin is zero.
18)	T	F	Every vector field which satisfies $\text{curl}(\vec{F}) = \vec{0}$ everywhere in the entire three dimensional space can be written as $\vec{F} = \text{grad}(f)$ for some scalar function f.
19)	T	F	Let $\vec{F}$ be a vector field and let S be an oriented surface $\vec{r}(u, v)$. Then $20 \int\int_S \vec{F} \cdot dS = \int\int_S \vec{G} \cdot dS$, where $\vec{G} = 20\vec{F}$.

20) | T | F |

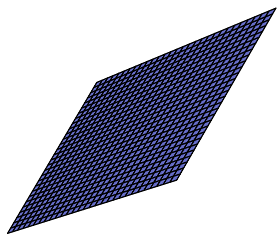
Consider the surface S given by the equation $z^2 = f(x, y)$. If $(x, y, z) = (x, y, \sqrt{f(x, y)})$ is a point on the surface with maximal distance from the origin, it is a local maximum of $g(x, y) = x^2 + y^2 + f(x, y)$.

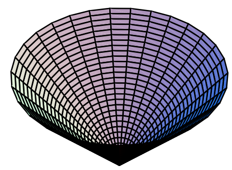
Problem 2) (10 points)

Match the parameterized surface formulas and pictures with the formulas for the implicit surfaces. No justifications are needed.

A)	$\vec{r}(u,v) = \langle 1+u, v, u+v \rangle$
B)	$\vec{r}(u,v) = \langle v\cos(u), v\sin(u), v \rangle$
C)	$\vec{r}(u,v) = \langle \cos(u)\sin(v), \sin(u)\sin(v), \cos(v) \rangle$
D)	$\vec{r}(u,v) = \langle u, v, u^2 - v^2 \rangle$
E)	$\vec{r}(u,v) = \langle v, \sin(u), \cos(u) \rangle$

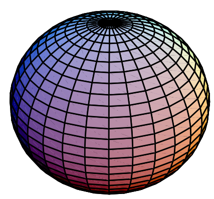
I



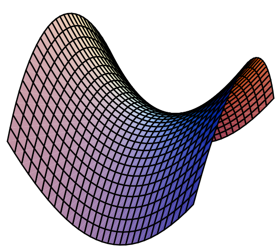


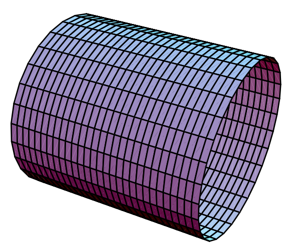
II

III



IV





V

Enter A),B),C),D),E) here	Enter I),II),III),IV),V) here	Equation
		$y^2 + z^2 = 1$
		$x + y - z = 1$
		$x^2 + y^2 + z^2 = 1$
		$x^2 + y^2 - z^2 = 0$
		$x^2 - y^2 - z = 0$

Problem 3) (10 points)

Match the formulas and theorems with their names. No justifications are needed.

N) $\int_a^b \int_c^d f(x,y)\, dydx = \int_c^d \int_a^b f(x,y)\, dxdy$
U) $f_{xy}(x,y) = f_{yx}(y,x)$
M) $\frac{\vec{v}\cdot\vec{w}}{|\vec{w}|^2}\vec{w}$
E) $\frac{\vec{v}\cdot\vec{w}}{|\vec{w}|}$
R) $\frac{d}{dt}f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$
A) $\vec{v}\cdot\vec{w} = |\vec{v}||\vec{w}|\cos(\alpha)$
L) $(\vec{u}\times\vec{v})\cdot\vec{w}$
F) $|\vec{v}\cdot\vec{w}| \leq |\vec{v}|\cdot|\vec{w}|$
I) $|\vec{v}|^2 + |\vec{w}|^2 = |\vec{v}-\vec{w}|^2$ if $\vec{v}\cdot\vec{w} = 0$
G) $|\vec{v}+\vec{w}| \leq |\vec{v}| + |\vec{w}|$

Enter letters here	Object or theorem
	Fubini theorem
	Clairaut theorem
	vector projection
	scalar projection
	chain rule
	dot product formula
	scalar triple product
	Cauchy-Schwarz inequality
	Pythagorean theorem
	Triangle inequality

Problem 4) (10 points)

Let L be the line $\vec{r}(t) = \langle t, 0, 0\rangle$. We are also given a point $Q = (3,3,0)$ in space.

a) (2 points) What is the distance $d((x,y,z),Q)$ between a general point (x,y,z) and Q?

b) (3 points) What is the distance $d((x,y,z),L)$ between the point (x,y,z) and the line L?

c) (3 points) Find the equation for the set C of all points (x,y,z) satisfying

$$d((x,y,z),Q) = d((x,y,z),L)\ .$$

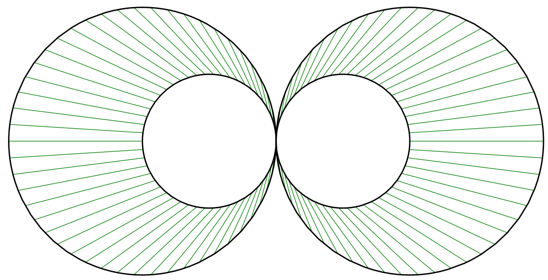
d) (2 points) Identify the surface.

Problem 5) (10 points)

Find the area of the region in the plane given in polar coordinates by

$$\{(r,\theta)\ |\ |\cos(\theta)| \leq r \leq 2|\cos(\theta)|, 0 \leq \theta < 2\pi\ \}\ .$$

The region is the shaded part in the figure.

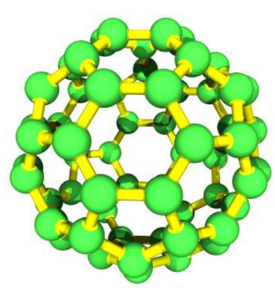


Problem 6) (10 points)

A microscopic bucky ball C60 is located on a gold surface. The surface produces the electric potential $f(x,y) = x^4 + y^4 - 2x^2 - 8y^2 + 5$.

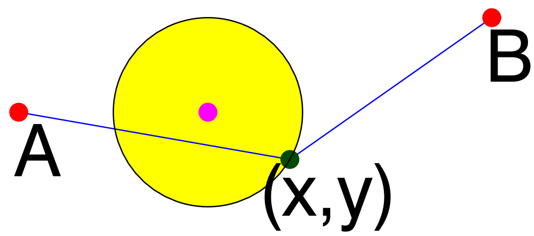
a) (7 points) Find all critical points of f and classify them.

b) (3 points) The fullerene will settle at a global minimum of $f(x,y)$. Find the global minima of the function $f(x,y)$.



Problem 7) (10 points)

A circular wheel with boundary $g(x,y) = x^2 + y^2 = 1$ has the boundary point (x,y) connected to two points $A = (-2,0)$ and $B = (3,1)$ by rubber bands. The potential energy at position (x, y) is by Hooks law equal to $f(x, y) = (x+2)^2 + y^2 + (x-3)^2 + (y-1)^2$, the sum of the squares of the distances to A and B. Our goal is to find the position (x, y) for which the energy is minimal. To find this position for which the wheel is at rest, minimize $f(x, y)$ under the constraint $g(x, y) = 1$.



Problem 8) (10 points)

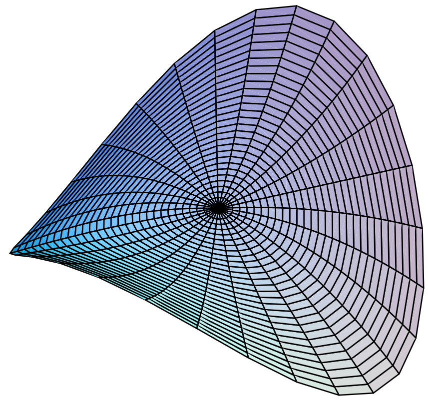
a) (5 points) Find the surface area of the parameterized surface

$$\vec{r}(u, v) = \langle u - v, u + v, uv \rangle$$

with $u^2 + v^2 \leq 1$.

b) (3 points) Find an implicit equation $g(x, y, z) = 0$ for this surface.

c) (2 points) What is the name of the surface?



Problem 9) (10 points)

a) (4 points) Find the tangent plane to the surface $f(x, y, z) = zx^5 + y^5 - z^5 = 1$ at the point $(1, 1, 1)$.

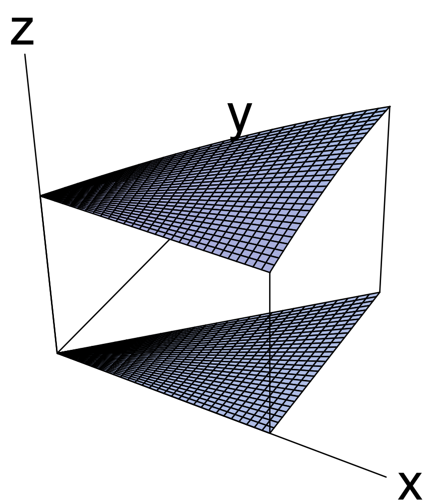
b) (3 points) Find the linearization $L(x, y, z)$ of $f(x, y, z)$ at at the point $(1, 1, 1)$.

c) (3 points) Near the point $(1, 1, 1)$, the surface can be written as a graph $z = g(x, y)$. Find the partial derivative $g_x(1, 1)$.

Problem 10) (10 points)

A tower E with base $0 \leq x \leq 1, 0 \leq y \leq x$ has a roof $f(x, y) = \sin(1 - y)/(1 - y)$. Find the volume of this solid. The solid is given in formulas by

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, 0 \leq z \leq f(x, y)\ \}\ .$$



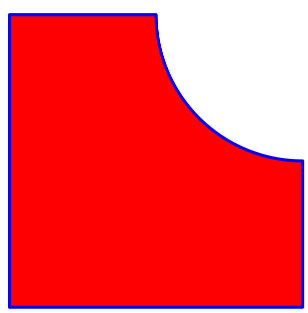
Problem 11) (10 points)

Let $\vec{F} = \langle y, 2x + \tan(\tan(y)) \rangle$ be a vector field in the plane and let C be the boundary of the region

$$G = \{0 \leq x \leq 2, 0 \leq y \leq 2, (x-2)^2 + (y-2)^2 \geq 1 \}$$

oriented counter clock-wise. Compute the line integral

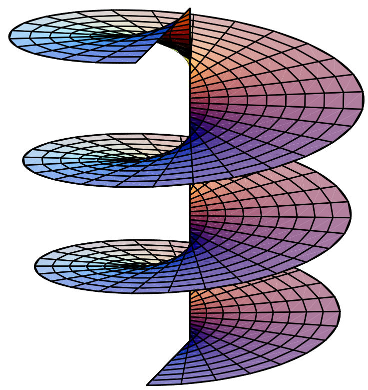
$$\int_C \vec{F} \cdot d\vec{r} .$$

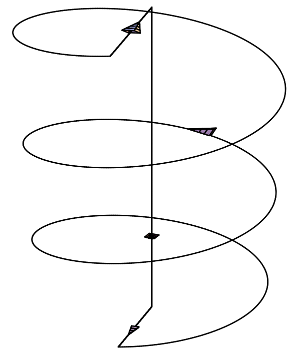


Problem 12) (10 points)

Let S be the surface of a turbine blade parameterized by $\vec{r}(s,t) = \langle s\cos(t), s\sin(t), t \rangle$ for $t \in [0, 6\pi]$ and $s \in [0,1]$. Let $\vec{F} = \text{curl}(\vec{G})$ denote the velocity field of the water velocity, where $\vec{G}(x,y,z) = \langle -y + (x^2+y^2-1), x + (x^2+y^2-1), (x^2+y^2-1) \rangle$. Compute the power of the turbine which is given by the flux of $\vec{F} = \text{curl}(\vec{G})$ through S.

Hint. The boundary C of the surface S consists of 4 paths:
$\vec{r_1}(t) = \langle \cos(t), \sin(t), t \rangle$, $t \in [0, 6\pi]$.
$\vec{r_2}(s) = \langle 1 - s, 0, 6\pi \rangle$, $s \in [0, 1]$.
$\vec{r_3}(t) = \langle 0, 0, 6\pi - t \rangle$, $t \in [0, 6\pi]$.
$\vec{r_4}(s) = \langle s, 0, 0 \rangle$, $s \in [0, 1]$.





Problem 13) (10 points)

Let $\vec{F}(x, y, z) = \langle z^2, -z^5 + z\sin(e^{\sin(x)}), (x^2 + y^2) \rangle$. Let S denote the part of the graph $z = 9 - x^2 - y^2$ lying above the xy-plane oriented so that the normal vector points upwards. Find the flux of $\vec{F}$ through the surface S.

Hint. You might also want to look at the surface $D = \{x^2 + y^2 \leq 9, z = 0\}$ lying in the xy-plane.

Problem 14) (10 points)

We are given two vector fields $\vec{F}(x,y,z) = \langle x, y, z \rangle$ and $\vec{G}(x,y,z) = \langle y, z, x \rangle$. We are also given a brezel surface S bounding a solid of volume 3 and a surface U which is part of S and has been left after cutting away a piece of the brezel. The surface U together with two discs D_1 and D_2 form a closed surface bounding a "cut brezel". The first surface S does not have a boundary and is oriented so that the normal vector points outwards. The second surface U has the same orientation than S and as a boundary the union C of two curves C_1 and C_2 which are oriented so that U is "to the left", when you look into the direction of the velocity vector and if the normal vector to U points "up".

a) (2 points) Compute $\int\int_S \vec{F}\, d\vec{S}$.
b) (2 points) Find $\int\int_S \vec{G}\, d\vec{S}$.
c) (2 points) Verify that the vector field $\vec{A} = \langle -xy, -yz, -zx \rangle$ is a **vector potential** of $\vec{G}$ meaning that $\vec{G} = \text{curl}(\vec{A})$.
d) (2 points) Express $\int_C \vec{A} \cdot \, \vec{dr}$ as a flux integral through U.
e) (2 points) Compute $\int_C \vec{F} \cdot \, \vec{dr}$.