Name:

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TTH 10 Gijs Heuts
TTH 11:30 Francesco Cavazzani
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- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-2 and 8, we need to see details of your computation.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F For a moving frame $(\vec{T}, \vec{N}, \vec{B})$, (remember that $\vec{T}$ is the unit tangent vector, $\vec{N}$ is the normal vector and $\vec{B}$ is the binormal vector), one always has $\vec{B} \cdot (\vec{T} \times \vec{N}) = 1$.

2) T F For any three points P, Q, R in space, $\vec{PQ} \times \vec{PR} = \vec{QP} \times \vec{RP}$

3) T F The triangle defined by the three points $(-1, 0, 2), (-4, 2, 1), (1, -1, 2)$ has a right angle.

4) T F The function $f(x, y, z) = x^2 + y^2 + z^2 / \sin(x^2 + y^2 + z^2)$ is continuous everywhere in space.

5) T F $\vec{u} \times \vec{u} = 0$ implies $\vec{u} = \vec{0}$.

6) T F The level curves $f(x, y) = 1$ and $f(x, y) = 2$ of a smooth function f never intersect.

7) T F For any vector $\vec{v}$, we have $\text{proj}_{\vec{i}}(\text{proj}_{\vec{j}}(\vec{v})) = \vec{0}$.

8) T F $(\vec{j} \times \vec{i}) \times \vec{i} = \vec{k} \times (\vec{i} \times \vec{k})$

9) T F If a parametrized curve $\vec{r}(t)$ lies in a plane and the velocity $\vec{r}\,'(t)$ is never zero, then the normal vector $\vec{N}(t)$ also lies in that plane.

10) T F The angle between $\vec{r}\,'(t)$ and $\vec{r}\,''(t)$ is always 90 degrees.

11) T F If $\vec{v}, \vec{w}$ are two nonzero vectors, then the projection vector $proj_{\vec{w}}(\vec{v})$ can be longer than $\vec{v}$.

12) T F A line intersects an ellipsoid in at most 2 distinct points.

13) T F For any vectors $\vec{v}$ and $\vec{w}$, the formula $(\vec{v} - \vec{w}) \cdot \vec{P}_{\vec{w}}(\vec{v}) = 0$ holds.

14) T F Let S be a plane normal to the vector $\vec{n}$, and let P and Q be points not on S. If $\vec{n} \cdot \vec{PQ} = 0$, then P and Q lie on the same side of S.

15) T F The vectors $\langle 2, 2, 1 \rangle$ and $\langle 1, 1, -4 \rangle$ are perpendicular.

16) T F $\|\vec{v} \times \vec{w}\| = \|v\| \|w\| \cos(\alpha)$, where α is the angle between $\vec{v}$ and $\vec{w}$.

17) T F The vector $\vec{i} \times (\vec{j} \times \vec{k})$ has length 1.

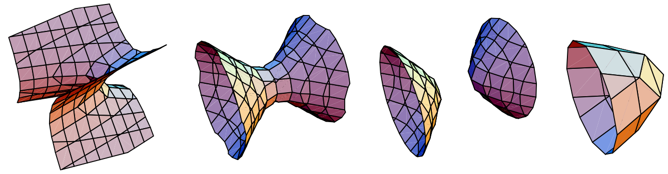
18) T F The distance between the z-axis and the line $x - 1 = y = 0$ is 1.

19) T F There is a quadric surface which both hyperbola and parabola appear as traces. Traces are intersections of the surface with the coordinate planes $x = 0$, $y = 0$, or $z = 0$.

20) T F The equation $x^2 + y^2 - z^2 = -1$ defines a one-sheeted hyperboloid.

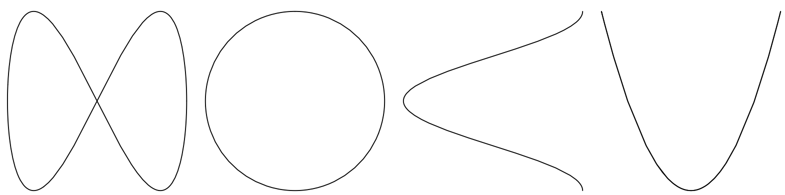
Problem 2) (10 points)

Match the equation with the pictures. No justifications are necessary in this problem.



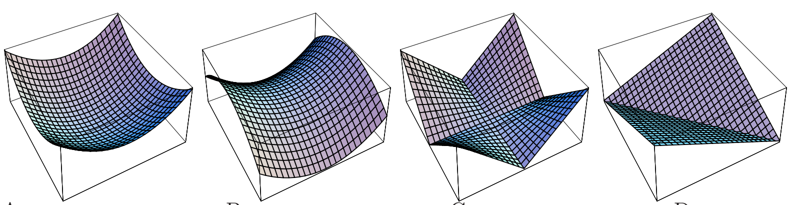
I II III IV

Enter I,II,III,IV here	Equation
	$x + y^2 - z^2 - 1 = 0$
	$-x^2 + y^2 + z^2 - 1 = 0$
	$-x^2 + y^2 + z^2 + 1 = 0$
	$-x + y^2 + z^2 + 1 = 0$



1 2 3 4

Enter 1,2,3,4 here	Equation
	$\langle \cos(t), \sin(t) \rangle$
	$\langle \cos(t), t \rangle$
	$\langle \cos(t), \cos^2(t) \rangle$
	$\langle \cos(t), \sin(2t) \rangle$



A B C D

Enter A,B,C,D here	Equation
	$f(x,y) = x^2 - y^2$
	$f(x,y) = \|x + y\|$
	$f(x,y) = x^2 + y^2$
	$f(x,y) = \|xy\|$

Problem 3) (10 points)

Imagine the planet Earth as the unit sphere in 3D space centered at the origin. An asteroid is approaching from the point $P = (0, 4, 3)$ along the path

$$\vec{r}(t) = \langle (4-t)\sin(t), (4-t)\cos(t), 3-t \rangle \ .$$

a) When and where will it first hit the Earth?

b) What velocity will it have at the impact?

Problem 4) (10 points)

Find the distance between the cylinder $x^2 + y^2 = 1$ and the line

$$L : \frac{x+2}{4} = \frac{y-1}{3} = \frac{z}{2} .$$

Problem 5) (10 points)

a) Find a parametrization $\vec{r}(t)$ of the line which is the intersection of the two planes

$$4x + 6y - z = 1$$

and

$$4x + z = 0 \ .$$

b) Find the point on the line which is closest to the origin.

Problem 6) (10 points)

Consider the parameterized curve

$$\vec{r}(t) = \langle e^t + e^{-t}, 2\cos(t), 2\sin(t) \rangle \ .$$

Find the arc length of this curve from $t = 0$ to $t = 4$.

Problem 7) (10 points)

The set of points P for which the distance from P to $A = (1, 2, 3)$ is equal to the distance from P to $B = (5, 8, 5)$ forms a plane S.

a) Find the equation $ax + by + cz = d$ of the plane S.

b) Find the distance from A to S.

Problem 8) (10 points)

The Swiss tennis player Roger Federer hits the ball at the point $\vec{r}(0) = (0, 0, 3)$. The initial velocity is $\vec{r}\,'(0) = \langle 100, 10, 13 \rangle$. The tennis ball experiences a constant acceleration $\vec{r}\,''(t) = \langle 2, 0, -32 \rangle$ which is due to the combined force of gravity and a constant wind in the x direction.

a) Where does the tennis ball hit the ground $z = 0$?

b) What is the z-component = (projection onto z vector) $proj_{\vec{k}}(\vec{r}\,'(t))$ of the ball velocity at the impact?

Problem 9) (10 points)

a) (4 points) Parameterize the intersection of the ellipsoid

$$\frac{x^2}{4} + \frac{(y-5)^2}{4} + \frac{z^2}{9} = 2$$

with the plane $z = 3$.

b) (3 points) Parametrize the ellipsoid itself in the form

$$\vec{r}(\theta, \phi) = ... \, .$$

c) (3 points) What is the curvature of the curve at the point $(2, 5, 3)$?

Hint. While you can use the curvature formula $\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$ you are also allowed to cite a fact which you know about the curvature.

Problem 10) (10 points)

Find an equation $ax + by + cz = d$ for the plane which has the property that $Q = (5, 4, 5)$ is the reflection of $P = (1, 2, 3)$ through that plane.