11/6/2012 SECOND HOURLY PRACTICE III — Math 21a, Fall 2012

Name:

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MWF 10 Joe Rabinoff
MWF 11 John Hall
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TTH 10 Gijs Heuts
TTH 11:30 Francesco Cavazzani
TTH 11:30 Andrew Cotton-Clay

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, and 5, we need to see **details** of your computation.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

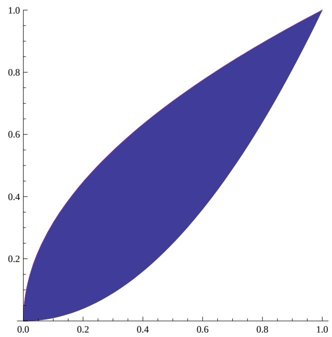
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False questions (20 points), no justifications needed

1)	T	F	Every function $f(x, y)$ of two variables has either a global minimum or a global maximum.
2)	T	F	The linearization of the function $f(x, y) = e^{x+3y}$ at $(0, 0)$ is $L(x, y) = 1 + x + 3y$.
3)	T	F	The function $f(x, y, z) = x^2 \cos(z) + x^3y^2z + (y-2)^3y^5$ satisfies the partial differential equation $f_{xyxzxy} = 12$.
4)	T	F	If $xe^z = y^2z$, then $\partial z/\partial x = e^z/(y^2 - xe^z)$.
5)	T	F	The function $\cos(x^2)\cos(y^2)$ has a local maximum at $(0, 0)$.
6)	T	F	The value of the double integral $\int_0^{\pi/4} \int_0^2 x^3 \cos(y) \; dxdy$ is the same as $(\int_0^2 x^3 \; dx)(\int_0^{\pi/4} \cos(y) \; dy)$.
7)	T	F	The gradient of $f(x, y)$ is always tangent to the level curves of f.
8)	T	F	If $f(x, y, z) = x - 2y + z$, then the largest possible directional derivative $D_{\vec{u}}f$ at any point in space is $\sqrt{6}$.
9)	T	F	$\int_0^1 \int_0^1 (x^2 + y^2) \; dxdy = \int_0^1 \int_0^1 r^3 \; drd\theta$.
10)	T	F	It is possible that the directional derivative $D_{\vec{v}}f$ is positive for all unit vectors $\vec{v}$.
11)	T	F	Using linearization of $f(x, y) = xy$ we can estimate $f(0.999, 1.01) \sim 1 - 0.001 + 0.01 = 1.009$.
12)	T	F	Given a curve $\vec{r}(t)$ on a surface $g(x, y, z) = -1$, then $\frac{d}{dt}g(\vec{r}(t)) < 0$.
13)	T	F	If $f(x, y)$ has a local minimum at $(0, 0)$ then it is possible that $f_{xy}(0, 0) > 0$.
14)	T	F	The function $f(x, y) = -x^8 - 2x^6 - y^8$ has a local minimum at $(0, 0)$.
15)	T	F	If $\vec{r}(t)$ is a curve in space and f is a function of three variables, then $\frac{d}{dt}f(\vec{r}(t)) = 0$ for $t = 0$ implies that $\vec{r}(0)$ is a critical point of $f(x, y, z)$.
16)	T	F	Let a, b, c be the number of saddle points, maxima and minima of a function $f(x, y)$. Then $a \leq b + c$.
17)	T	F	If $f(x, y)$ is a nonzero function of two variables and R is a region, then $\int\int_R f(x, y) \; dxdy$ is the volume under the graph of f and therefore a positive value.
18)	T	F	We extremize $f(x, y)$ under the constraint $g(x, y) = c$ and obtain a solution (x_0, y_0). If the Lagrange multiplier λ is positive, then the solution is a minimum.
19)	T	F	The tangent plane to a surface $f(x, y, z) = 1$ intersects the surface in exactly one point.
20)	T	F	Let $\vec{v}$ be a vector of length 1 in space. Given a function $f(x, y, z)$ of three variables. If (x_0, y_0, z_0) is a critical point of f, then it is a critical point of $g(x, y, z) = D_v f(x, y, z)$.

Problem 2) (10 points)

a) (6 points) Match the regions with the corresponding double integrals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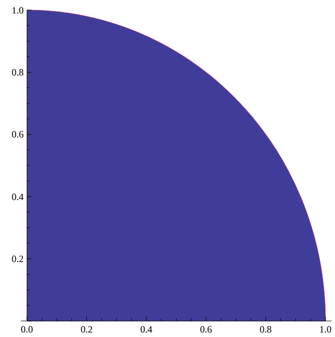
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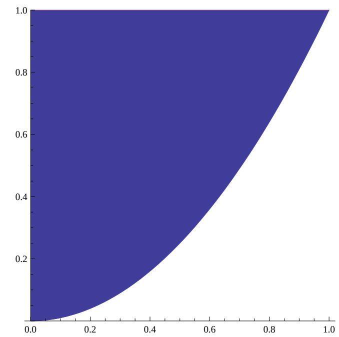
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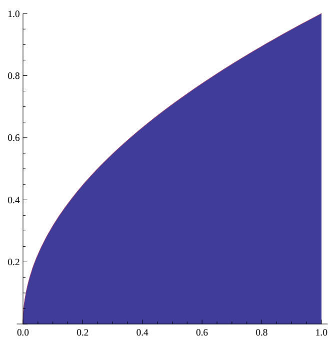
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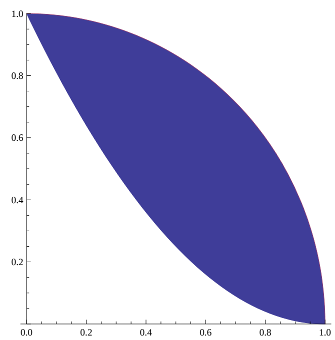
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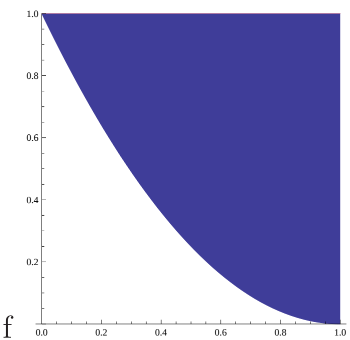

f

Enter a,b,c,d,e or f	Integral of $f(x,y)$
	$\int_0^1 \int_{x^2}^{\sqrt{x}} f(x,y)\, dydx$
	$\int_0^1 \int_0^{\sqrt{y}} f(x,y)\, dxdy$
	$\int_0^1 \int_{y^2}^1 f(x,y)\, dxdy$

Enter a,b,c,d,e or f	Integral of $f(x,y)$
	$\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)\, dydx$
	$\int_0^1 \int_{(1-x)^2}^1 f(x,y)\, dydx$
	$\int_0^1 \int_{(1-x)^2}^{\sqrt{1-x^2}} f(x,y)\, dydx$

b) (4 points) Match the PDE's with the names. No justifications are needed.

Enter A,B,C,D here	PDE
	$f_{xx} = -f_{yy}$
	$f_x = f_y$

Enter A,B,C,D here	PDE
	$f_{xx} = f_{yy}$
	$f_x = f_{yy}$

A) Wave equation	B) Heat equation	C) Transport equation	D) Laplace equation

Problem 3) (10 points)

a) (3 points) Find and classify all the critical points of $f(x, y) = xy - x$ on the plane.

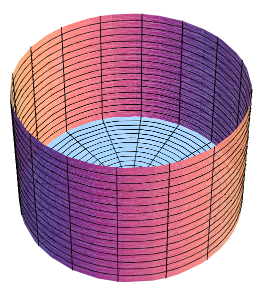
b) (2 points) Decide whether an absolute maximum or an absolute minimum of f exists on the plane R^2.

c) (3 points) Use the method of Lagrange multipliers to find the maximum and minimum of f on the boundary $x^2 + 4y^2 = 12$ of the elliptical region $G : x^2 + 4y^2 \leq 12$.

d) (2 points) Find the absolute maximum and absolute minimum of f on the region G given in c).

Problem 4) (10 points)

Find the cylindrical basket which is open on the top has has the largest volume for fixed area π. If x is the radius and y is the height, we have to extremize $f(x, y) = \pi x^2 y$ under the constraint $g(x, y) = 2\pi xy + \pi x^2 = \pi$. Use the method of Lagrange multipliers.

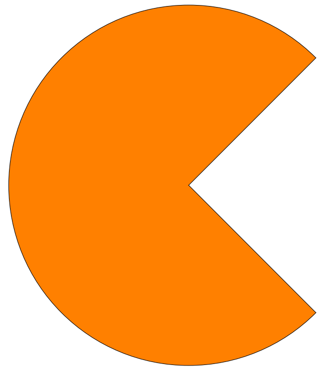


Problem 5) (10 points)

The Pac-Man region R is bounded by the lines $y = x, y = -x$ and the unit circle. The number

$$a = \frac{\int \int_R x \; dxdy}{\int \int_R 1 \; dxdy}$$

defines the point $C = (a, 0)$ called center of mass of the region. Find it.



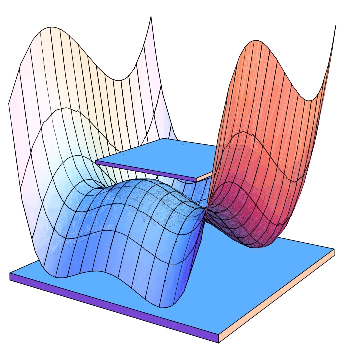
Problem 6) (10 points)

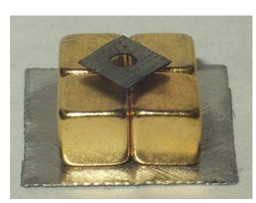
a) (5 points) Find the tangent plane to the surface $\sqrt{xyz} = 60$ at $(x, y, z) = (100, 36, 1)$.

b) (5 points) Estimate $\sqrt{100.1 * 36.1 * 0.999}$ using linear approximation. Here, for clarity reasons, we use $*$ for the usual multiplication for numbers.

Problem 7) (10 points)

Oliver got a diagmagnetic kit, where strong magnets produce a force field in which pyrolytic graphic flots. The gravitational field produces a well of the form $f(x, y) = x^4 + y^3 - 2x^2 - 3y$. Find all critical points of this function and classify them. Is there a global minimum?





Right picture credit: Wikipedia.

Problem 8) (10 points)

Let $f(x, y) = xy$.

a) (2 points) Find the direction of maximal increase at the point $(1, 1)$.

b) (3 points) Find the directional derivative at $(1, 1)$ in the direction $\langle 3/5, 4/5 \rangle$.

c) (2 points) The curve $\vec{r}(t) = \langle \sqrt{2}\sin(t), \sqrt{2}\cos(t) \rangle$ passes through the point $(1, 1)$ at some time t_0. Find $\frac{d}{dt} f(\vec{r}(t))$ at time t_0 directly.

d) (3 points) Find $\frac{d}{dt} f(\vec{r}(t))$ at time t_0 using the multivariable chain rule.

Problem 9) (10 points)

Integrate the function

$$f(x, y) = \frac{y^5 - 1}{y^{1/3} - y^{1/4}}$$

on the finite region bounded by the curves $y = x^3$ and $y = x^4$.

Problem 10) (10 points)

The main building of a mill has a cone shaped roof and cylindrical walls. If the cylinder has radius r, the height of the side wall is h and the height of the roof is h, then the volume is

$$V(h, r) = \pi r^2 h + h\pi r^2/3 = (4\pi/3)hr^2$$

and assume the cost of the building is

$$A(h, r) = \pi r^2 + 2\pi r h + \pi 2r^2 = \pi(3r^2 + 2rh)$$

which is the area of the ground plus the area of the wall plus $2\pi r h$, the cost for the roof. For fixed volume $V(h, r) = 4\pi/3$, minimize the cost $A(h, r)$ using the Lagrange multiplier method.