Name:

MWF 9 Oliver Knill
MWF 9 Chao Li
MWF 10 Gijs Heuts
MWF 10 Adrian Zahariuc
MWF 10 Yihang Zhu
MWF 11 Peter Garfield
MWF 11 Matthew Woolf
MWF 12 Charmaine Sia
MWF 12 Steve Wang
MWF 14 Mike Hopkins
TTH 10 Oliver Knill
TTH 10 Francesco Cavazzani
TTH 11:30 Kate Penner
TTH 11:30 Francesco Cavazzani

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3,8, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications are needed.

1) T F The vector $\langle 0, 6/10, 8/10 \rangle$ is a direction = unit vector.

2) T F Two nonzero vectors $\vec{v}$ and $\vec{w}$ are perpendicular if $\vec{v} \times \vec{w} = \vec{0}$.

3) T F For any vectors $\vec{u}$ and $\vec{v}$, we must have $\vec{v} \cdot \text{Proj}_{\vec{u}}\vec{v} = \vec{u} \cdot \text{Proj}_{\vec{v}}\vec{u}$.

4) T F The plane parametrized by $\vec{r}(t,s) = \langle t, s, 1 \rangle$ is the same as $z = 1$.

5) T F The surface $x^2 + y^2 - 2y - z^2 = 0$ is a cone.

6) T F The curvature of the helix $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$ at any t is less than 1.

7) T F The volume of a parallelepiped generated by the vectors $\vec{u}, \vec{v}, \vec{w}$ is equal to $|(\vec{u} \times \vec{v}) \cdot \vec{w}|$.

8) T F If a curve in space is parametrized by $\vec{r}(t)$ with $0 \leq t \leq 1$, then the same curve in the opposite direction can be parametrized by $\vec{r}(1-t)$ with $0 \leq t \leq 1$.

9) T F The two-sheeted hyperboloid $x^2 + y^2 - z^2 = -1$ separates space into regions. The points $(3, 4, 6)$ and $(5, 12, -14)$ lie in the same region.

10) T F Given two vectors $\vec{u}$ and $\vec{v}$ which are perpendicular. Then $\text{Proj}_{\vec{u}}(\text{Proj}_{\vec{v}}\vec{w}) = \vec{0}$ for any vector $\vec{w}$.

11) T F The velocity vector $\vec{r}\,'(t)$ is always perpendicular to the curve.

12) T F If a point P with cylindrical coordinates (r, θ, z) and spherical coordinates (ρ, θ, ϕ) has the property that $r = \rho$, then it must be on the xy plane.

13) T F The curvature of a circle of radius 3 is 1/3.

14) T F The triple scalar product satisfies $\vec{u} \cdot (\vec{v} \times \vec{w}) \leq |\vec{u}||\vec{v}||\vec{w}|$.

15) T F If the dot product between two vectors is positive, then the two vectors form an acute angle.

16) T F The surface given in cylindrical coordinates as $z^2 + r^2 = 1$ is a sphere.

17) T F The arc length of the curve $\langle \sin(t), \cos(t) \rangle$ from $t = 0$ to $t = 1$ is equal to 1.

18) T F The curve $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$ hits the plane $z = 0$ at a right angle.

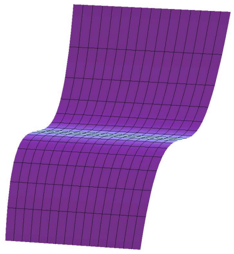
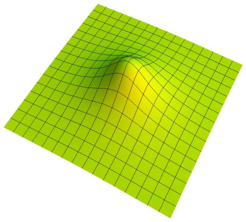
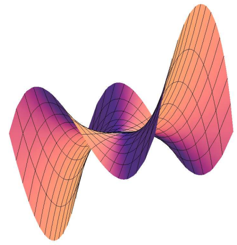
19) T F The parametrized curve $\langle 0, 7\cos(1+t), 3\sin(1+t) \rangle$ is an ellipse.

20) T F $\vec{u} \times (\vec{v} \times \vec{u}) = \vec{0}$ for all vectors $\vec{u}, \vec{v}$.

Total

Problem 2) (10 points) No justifications are needed here.

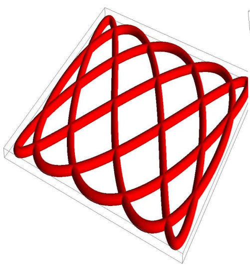
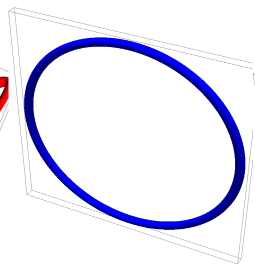
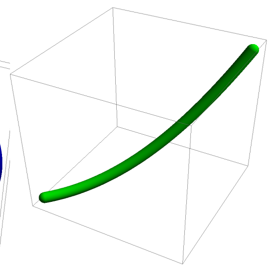
a) (2 points) Match the graphs of the functions $f(x, y)$. Enter O, if there is no match.

I II III

Function $f(x, y) =$	Enter O,I,II or III
$x^3 - xy^2$	
y^3	
$1/(1 + x^2 + y^2)$	
$x^4 + y^4$	

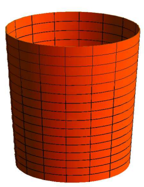
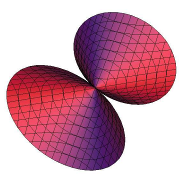
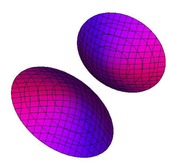
b) (2 points) Match the plane curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.

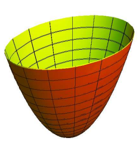
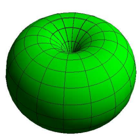
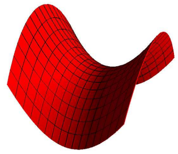
I II III

Parametrization $\vec{r}(t) =$	O, I,II,III
$\vec{r}(t) = \langle \cos(3t), \sin(5t), 0 \rangle$	
$\vec{r}(t) = \langle t, t, t^2 \rangle$	
$\vec{r}(t) = \langle \cos(t), 0, \sin(t) \rangle$	
$\vec{r}(t) = \langle \sin(t), \sin(t), \sin(t) \rangle$	

c) (4 points) Match the surfaces to the pictures. There is an exact match here.

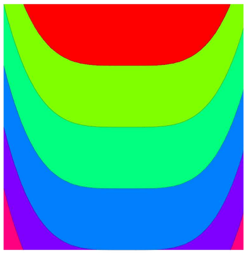
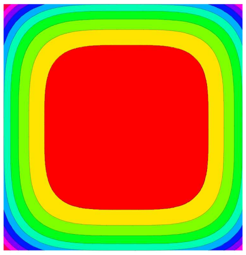
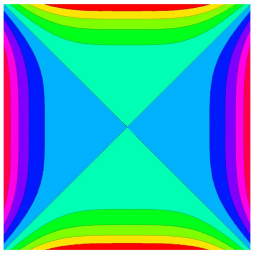
I II III

IV V VI

Description	I,II,III,IV,V,VI
$\langle 2u\cos(v), 4u\sin(v), u^2 \rangle$	
$\langle u^3, v^3, u^6 - v^6 \rangle$	
$\rho = \sin(\phi)$	
$r = 1$	
$x^2 - y^2 + z^2 = -1$	
$x^2 = y^2 - z^2$	

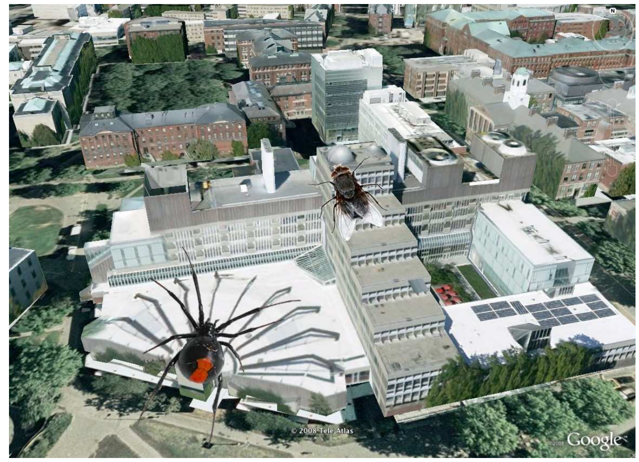
d) (2 points) Match the contour maps for $f(x, y)$. Enter O if no match.

I II III

function $f(x, y) =$	O,I,II,III
$f(x, y) = x^4 + y^4$	
$f(x, y) = x^4 - y^4$	
$f(x, y) = x - y$	
$f(x, y) = x^4 - y$	

Problem 3) (10 points)



The front roof line of the "spider" on the Harvard lecture halls forms a line

$$\vec{r}(t) = \langle 1+t, 2+t, 1 \rangle \ .$$

On top of the telescope sits a fly at the point $P = (0,0,10)$. Find the distance of P to the line.

Problem 4) (10 points)

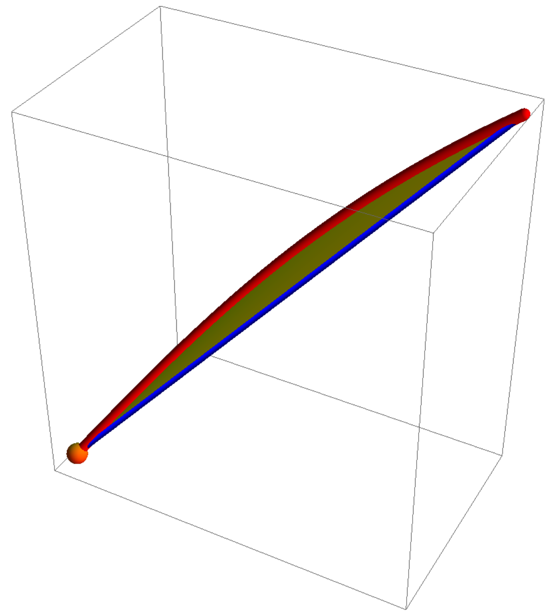
The kinect sensor can be used to scan objects. An infrared laser is used to measure distances in the horizontal plane.

a) (2 points) Find an equation which tells that a point $P = (x,y)$ has distance 5 from the sensor $(0,-1)$.

b) (2 points) Find an equation which tells that a point $P = (x,y)$ has distance 4 from the sensor $(0,1)$.

c) (6 points) Assume we know that P has distance 5 from $(0,-1)$ and distance 4 from $(0,1)$. Where is this point (x,y) if we assume that it has a positive x-coordinate?

Problem 5) (10 points)



a) (6 points) Given $\vec{r}(t) = \langle t + t^3/3, \arctan(t), \sqrt{2}t \rangle$. Find the arc length from $t = 0$ to $t = 1$.

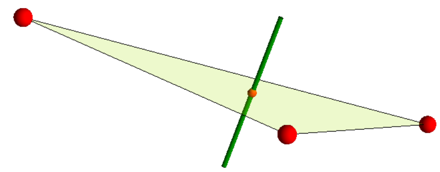
b) (4 points) Compute the vector integral

$$\int_0^1 \vec{r}\,'(t)\; dt$$

by integrating coordinate by coordinate. Verify that the length of this vector agrees with the arc length of the straight line connecting $\vec{r}(0)$ with $\vec{r}(1)$.

Problem 6) (10 points)

Given four points $A = (1, 2, 1), B = (1, 0, 1), C = (0, 1, 1), D = (1, 1, 2)$.



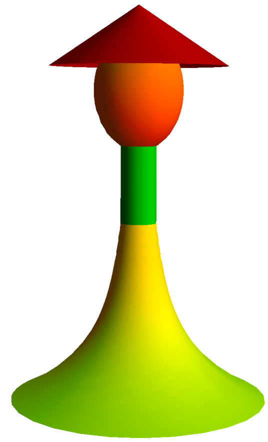
a) (4 points) Find an equation $ax + by + cz = d$ for the plane which contains A, B, C.
b) (3 points) Parametrize the line L which passes through D perpendicular to the plane ABC.
c) (3 points) Where does L hit the plane through A, B, C?

Problem 7) (10 points)

British stuntman **Gary Connery** made aviation history last year by becoming the first skydiver to land without parachute. He landed in 18000 boxes. Assume he started with an initial velocity $\langle 0, 100, 0 \rangle$ from the initial point $\langle 0, 0, 800 \rangle$. He was exposed to an acceleration $\vec{r}\,''(t) = \langle 0, 0, -10 + t \rangle$. Where is his location at time t=6?

Problem 8) (10 points)

We parametrize the queen in a fancy chess set. It consists of 5 surfaces. Parametrize them. You do not have to give bounds for the parameters. In each case, just give an answer of the form $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$ without further explanations.



a) (2 points) "hat" Cone $x^2 + y^2 = (1-z)^2$.

b) (2 points) "head" Sphere $x^2 + y^2 + (z+1/2)^2 = 1$.

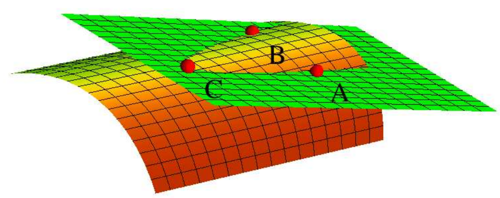
c) (2 points) "neck" Cylinder $x^2 + y^2 = 1/4$.

d) (2 points) "robe" Hyperboloid $x^2 + y^2 - (z+4)^2 = 1$.

e) (2 points) "floor" Plane $z = -8$

Problem 9) (10 points)

We are given a surface parametrized as $\vec{r}(u,v) = \langle u+v, u^2, v \rangle$.



a) (2 points) Locate the points $A = \vec{r}(1,2)$, $B = \vec{r}(-1,2)$ and $C = \vec{r}(0,0)$.

b) (4 points) Parametrize the plane through A, B, C.

c) (4 points) Find the area of the triangle with vertices A, B, C.