Name:

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MWF 10 Gijs Heuts
MWF 10 Yu-Wen Hsu
MWF 10 Yong-Suk Moon
MWF 11 Rosalie Belanger-Rioux
MWF 11 Gijs Heuts
MWF 11 Siu-Cheong Lau
MWF 12 Erick Knight
MWF 12 Kate Penner
TTH 10 Peter Smillie
TTH 10 Jeff Kuan
TTH 10 Yi Xie
TTH 11:30 Jeff Kuan
TTH 11:30 Jameel Al-Aidroos

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False (TF) questions (20 points)

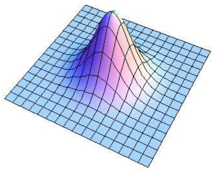
Mark for each of the 20 questions the correct letter. No justifications are needed.

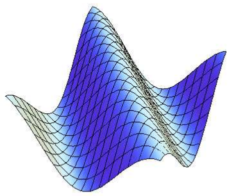
1)	T	F	If two planes $ax + by + cz = d$ and $Ax + By + Cz = D$ are parallel then $a = A, b = B$, and $c = C$.
2)	T	F	The point $(x, y, z) = (1, 1, \sqrt{2})$ has the spherical coordinates $(\rho, \theta, \phi) = (2, \pi/4, \pi/4)$.
3)	T	F	Every point on the parametric curve $\vec{r}(t) = \langle t, t^2, -t \rangle$ lies on the surface $xz + y = 0$.
4)	T	F	The two surfaces $f(x, y, z) = 3$ and $f(x, y, z) = 5$ of the function $f(x, y, z) = 2x^2 + y^3 + z^4$ do not intersect at any point in space.
5)	T	F	$\vec{u} \times \vec{i}$ and $\vec{u} \times \vec{j}$ are perpendicular for all vectors $\vec{u}$.
6)	T	F	If $\vec{u}$ and $\vec{v}$ are parallel (remember that this means $\vec{u} = \lambda\vec{v}$ for some real λ) then $\vec{u} \cdot \vec{v} \geq \lvert\vec{u} \times \vec{v}\rvert$.
7)	T	F	If a surface has the property that all intersections with the planes $y =$ *constant* are straight lines, then the surface is a plane.
8)	T	F	For any non-zero vectors $\vec{u}$ and $\vec{w}$, we must have $\text{Proj}_{\vec{u}}\vec{w} = -\text{Proj}_{\vec{w}}\vec{u}$.
9)	T	F	In the parametric surface $\vec{r}(s, t) = \langle \sqrt{1 + e^t} \cos(s) , \sqrt{1 + e^t} \sin(s) , t \rangle$ the grid curves with constant s are ellipses.
10)	T	F	There is a vector $\vec{v}$ with the property that $\vec{v} \times \langle 1, 1, 1 \rangle = \langle 0, 0, 1 \rangle$.
11)	T	F	We can assign a value $f(0, 0)$ such that the function $f(x, y) = (x^3+y^3)/(x^2+y^2)$ is continuous at $(0, 0)$.
12)	T	F	The curvature of a curves $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ and $\vec{R}(t) = \langle t^2, t^4, t^6 \rangle$ are the same at $t = 1$.
13)	T	F	The curve given in spherical coordinates as $\phi = \pi/2, \rho = \pi/2$ is a circle.
14)	T	F	Two nonparallel planes with normal vectors $\vec{n}, \vec{m}$ intersect in a line parallel to $\vec{n} \times \vec{m}$.
15)	T	F	If $f(x, y) = x^3/3 - y^2$, then the graph of the function $f(x, y)$ is called an elliptic paraboloid.
16)	T	F	The equation $\rho \cos(\theta) \sin(\phi) = 2$ in spherical coordinates defines a plane.
17)	T	F	The vector $\langle 3, -2 \rangle$ in the two dimensional plane is perpendicular to the line $3x - 2y = 7$.
18)	T	F	The volume of the parallelepiped spanned by the vectors $\langle 1, 0, 0 \rangle, \langle 0, 2, 0 \rangle$ and $\langle 1, 1, 1 \rangle$ is 2.
19)	T	F	If $\vec{r}(t)$ is a curve and $\lvert\vec{r}\,'(t)\rvert > 0$ and $\lvert\vec{T}'\rvert > 0$, we have $\vec{T}(t) \cdot (\vec{N}(t) \times \vec{B}(t)) = 1$.
20)	T	F	The arc lengths of $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ and $\vec{R}(t) = \langle t^2, t^4, t^6 \rangle$ are the same for $0 \leq t \leq 1$.

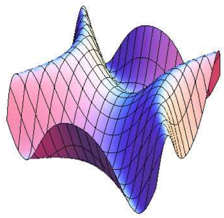
Total

Problem 2) (10 points)

a) (2 points) Match the graphs $z = f(x, y)$ with the functions. Enter O, if there is no match. In each of the problems a) - d), each entry O,I,II,III appears exactly once.

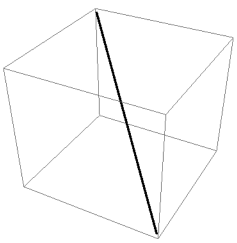

I

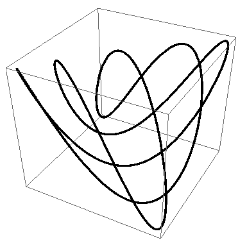

II

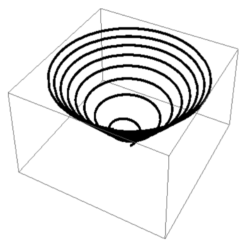

III

Function $f(x, y) =$	O,I,II or III
$e^{-x^2-y^2}$	
$\cos(x + y)$	
$\sin(x^2 - y^2)$	
$x^4 + y^4$	

b) (3 points) Match the space curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.

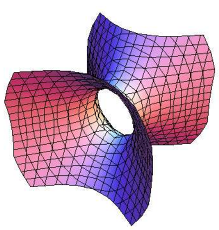

I

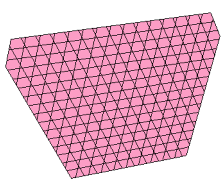

II


III

Parametrization $\vec{r}(t) =$	O, I,II,III
$\langle 1 + t, 1 - t, t\rangle$	
$\langle t\cos(t^2), t\sin(t^2), t\rangle$	
$\langle t, t, \sin(t^3)\rangle$	
$\langle \cos(3t), \sin(2t), \sin(5t)\rangle$	

c) (2 points) Match the functions g with the level surface $g(x, y, z) = 1$. Enter O, where no match.


I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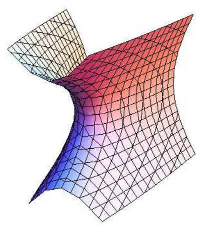
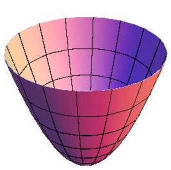
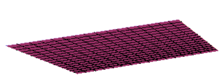
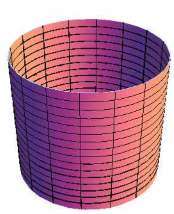

III

$g(x, y, z) =$	O, I,II,III
$(x - 1)^2 - y^2 + z^2 = 1$	
$(x - 1)^2 + y + z^2 = 1$	
$(x - 1) + y + z = 1$	
$(x - 1)^2 - y - z^2 = 1$	

d) (3 points) Match the surface with the parametrization. Enter O, where no match.


I

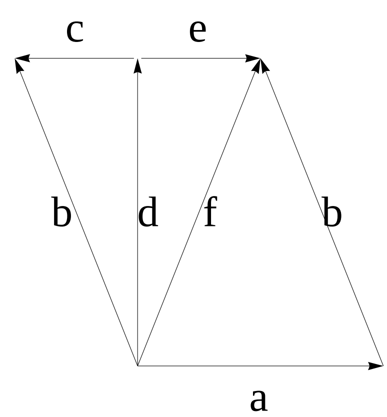

II


III

Parametriztion $\vec{r}(s, t) =$	O,I,II,III
$\langle s\cos(t), s\sin(t), s^2\rangle$	
$\langle t - 1, s, s + t\rangle$	
$\langle \cos(t), \sin(t), s\rangle$	
$\langle s\cos(t), s\sin(t), s^2\sin(t)\rangle$	

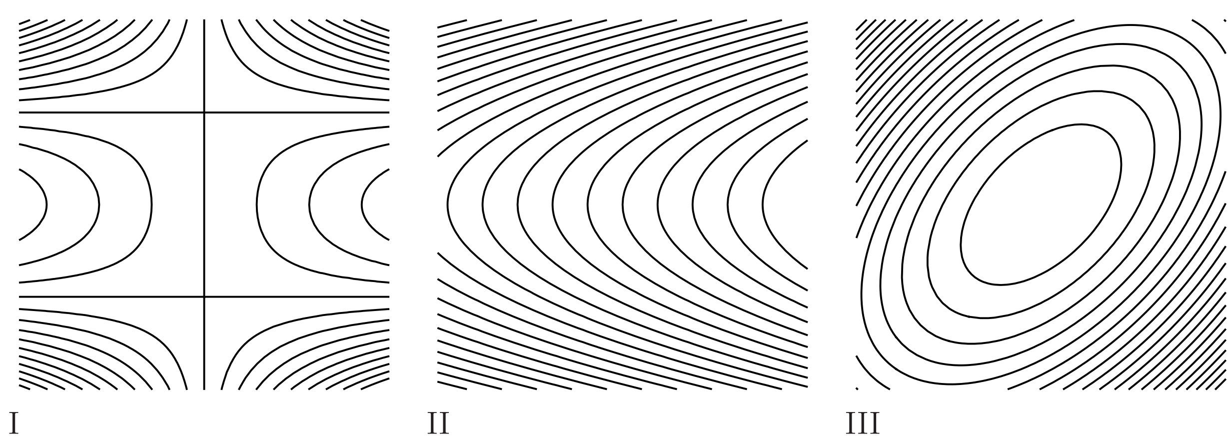
Problem 3) (10 points)

a) (7 points) Each of the vectors $a, b, c, d, e, f, 0$ (written without arrows for clarity) will appear in the blanks exactly once. As the picture indicates, you know $d \cdot e = d \cdot c = 0$.



the vector	is equal to
$\text{Proj}_d f$	
$f - d$	
$-2c$	
$d - c$	
$-e$	
$\text{Proj}_d e$	
$d + c$	

b) (3 points) Match the contour maps with the functions



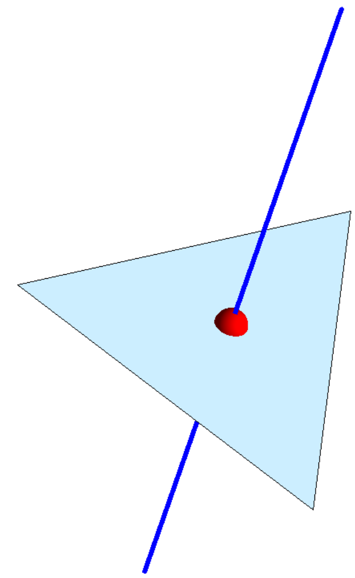
Function $f(x, y) =$	Enter O,I,II or III
$y - x$	
$(y^2 - 1)x$	
$y^2 + x^2 - xy$	
$y^2 - x$	

Problem 4) (10 points)

a) (4 points) The **center** of the triangle $A = (3,2,1), B = (1,1,1), C = (2,0,4)$ is the point $P = (A+B+C)/3 = (2,1,2)$. Find the line L perpendicular to the plane which contains A, B, C and which goes through P.

b) (3 points) Find the equation of the plane through A, B, C.

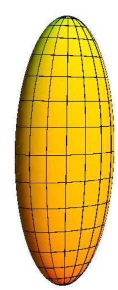
c) (3 points) Find the area of the triangle ABC.



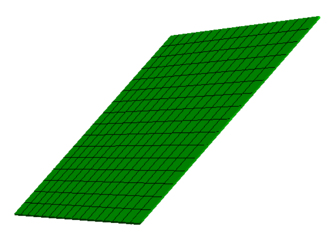
Problem 5) (10 points)

Complete the parametrizations:

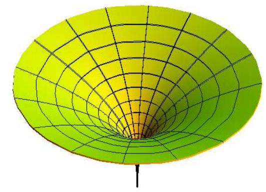
a) (3 points) $\vec{r}(u,v) = \langle 2 + 3\cos(u)\sin(v), 3 + \sin(u)\sin(v), \boxed{} \rangle$ parametrizes the ellipsoid $(x-2)^2/9 + (y-3)^2 + (z-5)^2/16 = 1$.



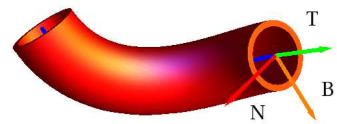
b) (2 points) $\vec{r}(u,v) = \langle u, v, \boxed{} \rangle$ parametrizes the plane $x + y + z = 1$.



c) (3 points) $\vec{r}(u,v) = \langle v^3\cos(u), \boxed{}, v\rangle$ parametrizes the surface of revolution $x^2+y^2=z^6$.



d) (2 points) $\vec{r}(u,v) = \vec{r}(v) + \cos(u)\vec{N}(v) + \sin(u)\boxed{}$ parametrizes a tube around a curve $\vec{r}(v)$ which has unit tangent vector $\vec{T}(v)$, normal vector $\vec{N}(v)$ and binormal vector $\vec{B}(v)$.

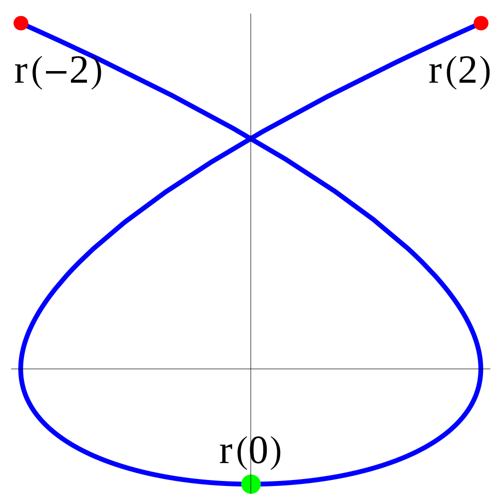


Problem 6) (10 points)

We look at the parametrized curve

$$\vec{r}(t) = \langle \frac{t^3}{3} - t, t^2-1, 0\rangle$$

whose image you see in the picture showing it in the xy plane for $-2 \leq t \leq 2$.



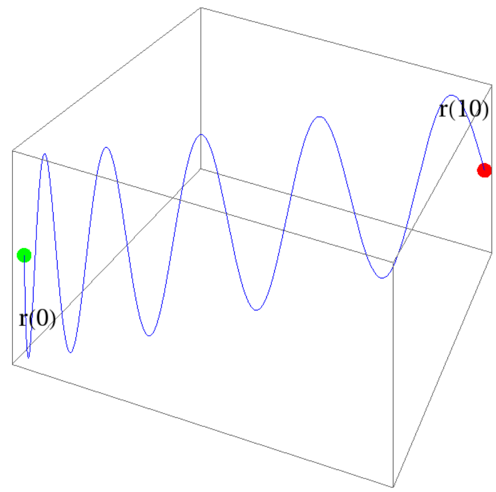
a) (3 points) Find the velocity $\vec{r}\,'(t)$, the acceleration $\vec{r}\,''(t)$ and speed $|\vec{r}\,'(t)|$.

b) (2 points) Evaluate this at $t = 0$ to get $\vec{r}\,'(0), \vec{r}\,''(0)$ and $|\vec{r}\,'(0)|$.

c) (2 points) Find the curvature $|\vec{r}\,'(0) \times \vec{r}\,''(0)|/|\vec{r}\,'(0)|^3$ at $(0,-1,0)$.

d) (3 points) Find the arc length of the curve $\vec{r}(t)$ from $-2 \leq t \leq 2$.

Problem 7) (10 points)

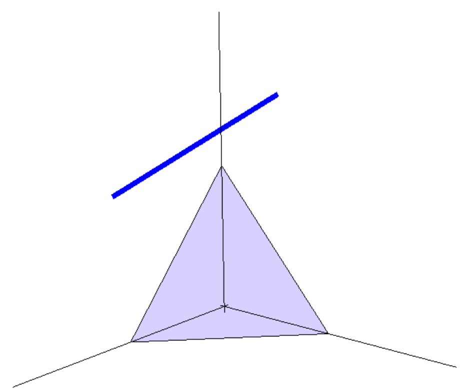


a) (4 points) We know $\vec{r}\,''(t) = \langle 1, 2, \pi^2 \sin(\pi t) \rangle$ and the initial velocity $\vec{r}\,'(0) = \langle 1, 0, -\pi \rangle$. Find $\vec{r}\,'(t)$.

b) (3 points) Assume we know also $\vec{r}(0) = \langle 0, 0, 10 \rangle$. Find $\vec{r}(10)$.

c) (3 points) What is the projection of $\vec{r}\ '(10)$ onto $\langle 1, 1, 0 \rangle$?

Problem 8) (10 points)

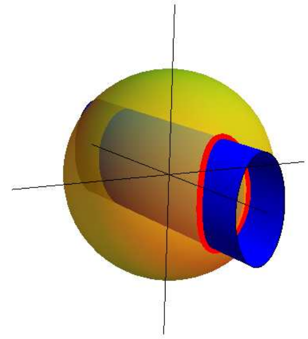


a) (5 points) Find the distance between the plane $x + y + z = 1$ and the line

$$x - 1 = \frac{(y-1)}{-2} = z - 1$$

which is parallel to the plane.
(You do not have to check that it is parallel).

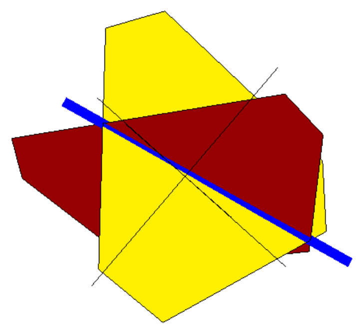
b) (5 points) The intersection of the cylinder $4x^2+z^2 = 1$ with the sphere centered at $(0,0,0)$ with radius $\rho = \sqrt{2}$ cuts out two curves. Parametrize the curve which contains the point $(0,1,1)$.



Problem 9) (10 points)

a) (5 points) Find a parametrization of the intersection line L of the two planes

$$\begin{aligned} 2x - 2y + z &= 1 \ , \\ x + y + z &= 1 \ . \end{aligned}$$



b) (5 points) Find the symmetric equation for the line M parallel to the line L computed in a) which passes through $(1,2,3)$.

Problem 10) (10 points)

a) (5 points) What is the area of the triangle through the points $A = (1, 1, 1)$ and $B = (0, 1, 0)$ and $C = (1, 2, 4)$.

b) (5 points) Find the volume of the prism which has the triangle T as base as well as a by $\vec{v} = \langle 0, 1, 1 \rangle$ translated triangle as roof.