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TTH 10 Yi Xie
TTH 11:30 Jeff Kuan
TTH 11:30 Jameel Al-Aidroos

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

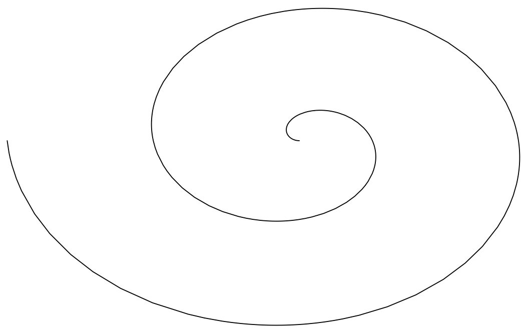
Problem 1) TF questions (20 points) No justifications needed

1)	T	F	The length of the sum of two vectors is always the sum of the length of the vectors.
2)	T	F	For any three vectors, $\vec{v} \times (\vec{w} + \vec{u}) = \vec{w} \times \vec{v} + \vec{u} \times \vec{v}$.
3)	T	F	The set of points which satisfy $x^2 + 2x + y^2 - z^2 = 0$ is a cone.
4)	T	F	The surface $\vec{r}(u,v) = \langle \cos(u^2)\sin(v^2), \sin(u^2)\sin(v^2), \cos(v^2) \rangle$ with $0 \leq u < \sqrt{2\pi}, 0 \leq v \leq \sqrt{\pi}$ is a sphere.
5)	T	F	If P, Q, R are 3 different points in space that don't lie in a line, then $\vec{PQ} \times \vec{RQ}$ is a vector orthogonal to the plane containing P, Q, R.
6)	T	F	The line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ hits the plane $2x + 3y + 4z = 9$ at a right angle.
7)	T	F	The function $f(x,y) = \sin(xy)/y$ is continuous everywhere.
8)	T	F	For any two vectors, $\vec{v} \times \vec{w} = \vec{w} \times \vec{v}$.
9)	T	F	If $\lvert \vec{v} \times \vec{w} \rvert = 0$ for all vectors $\vec{w}$, then $\vec{v} = \vec{0}$.
10)	T	F	If $\vec{u}$ and $\vec{v}$ are orthogonal vectors, then $(\vec{u} \times \vec{v}) \times \vec{u}$ is parallel to $\vec{v}$.
11)	T	F	Every vector contained in the plane $x + y + z = 1$ is parallel to the vector $\langle 1, 1, 1 \rangle$.
12)	T	F	The sphere can in cylindrical coordinates described as $r^2 = 1 - z^2$.
13)	T	F	The curvature of the curve $2\vec{r}(4t)$ at $t = 0$ is twice the curvature of the curve $\vec{r}(t)$ at $t = 0$.
14)	T	F	The set of points which satisfy $x^2 - 2y^2 - 3z^2 = 0$ form an ellipsoid.
15)	T	F	If $\vec{v} \times \vec{w} = (0,0,0)$, then $\vec{v} = \vec{w}$.
16)	T	F	Every vector contained in the line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ is parallel to the vector $\langle 1, 1, 1 \rangle$.
17)	T	F	Two nonzero vectors are parallel if and only if their cross product is $\vec{0}$.
18)	T	F	The vector $\vec{u} \times (\vec{v} \times \vec{w})$ is always in the same plane together with $\vec{v}$ and $\vec{w}$.
19)	T	F	The line $\vec{r}(t) = \langle 1 + 2t, 1 + 2t, 1 - 4t \rangle$ hits the plane $x + y + z = 9$ at a right angle.
20)	T	F	The intersection of the ellipsoid $x^2/3 + y^2/4 + z^2/3 = 1$ with the plane $y = 1$ is a circle.

Problem 2a) (3 points)

Match the curves with their parametric definitions.

I



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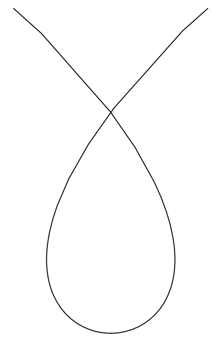

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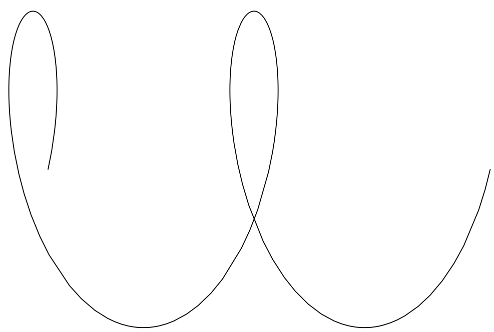

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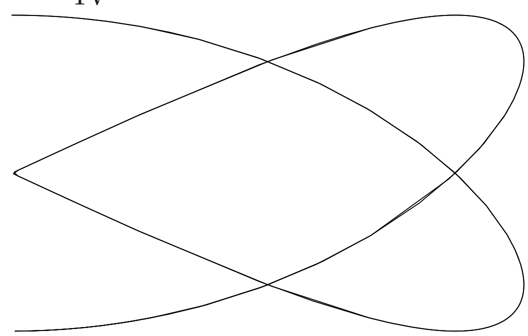

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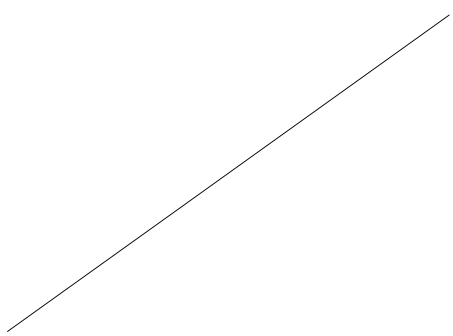

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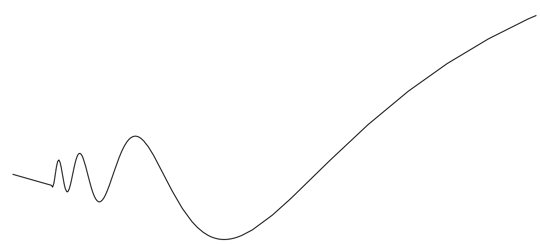
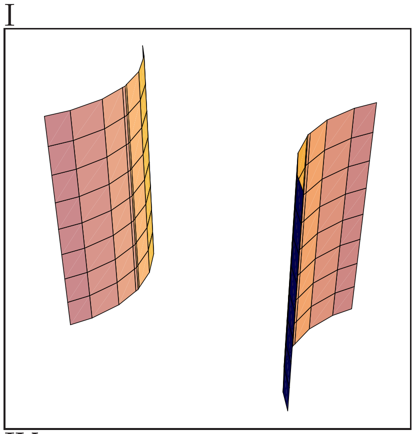


Enter I,II,III,IV,V or VI here	Parametric equation for the curve
	$\vec{r}(t) = \langle t, \sin(1/t)t \rangle$
	$\vec{r}(t) = \langle t^3 - t, t^2 \rangle$
	$\vec{r}(t) = \langle t + \cos(2t), \sin(2t) \rangle$
	$\vec{r}(t) = \langle \lvert \sin(2t) \rvert, \cos(3t) \rangle$
	$\vec{r}(t) = \langle 1 + t, 5 + 3t \rangle$
	$\vec{r}(t) = \langle -t\cos(t), 2t\sin(t) \rangle$

Problem 2b) (3 points)

Match the equations with the surfaces.

I



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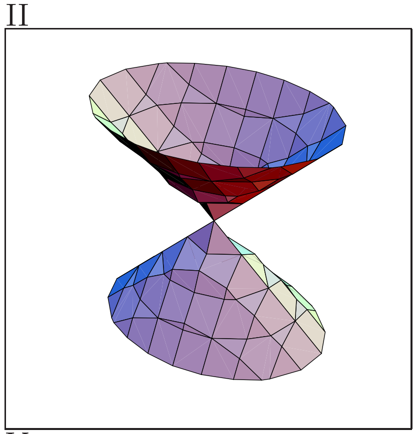

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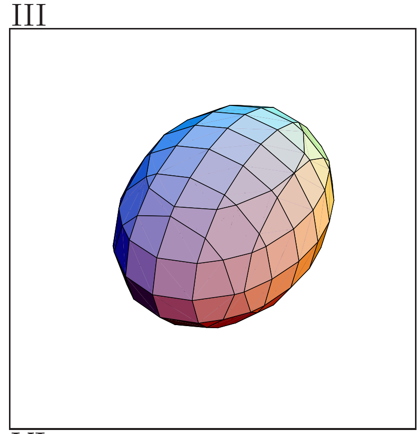

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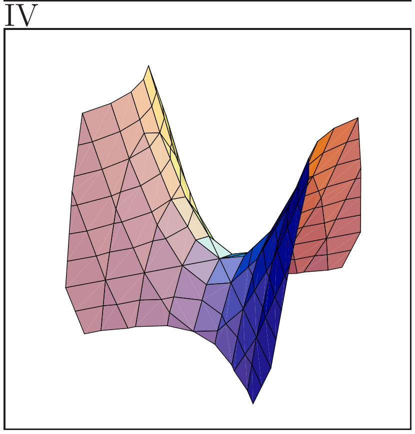

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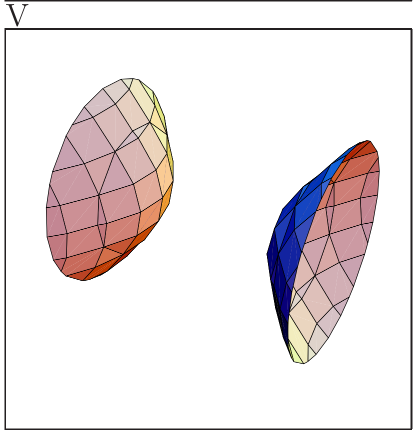

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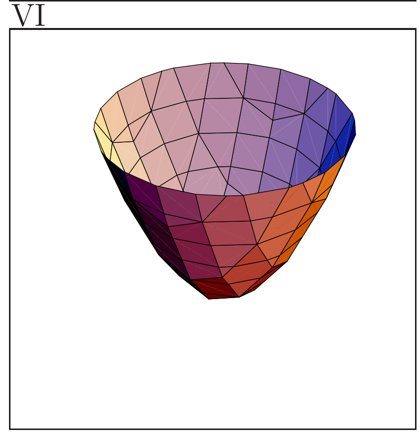
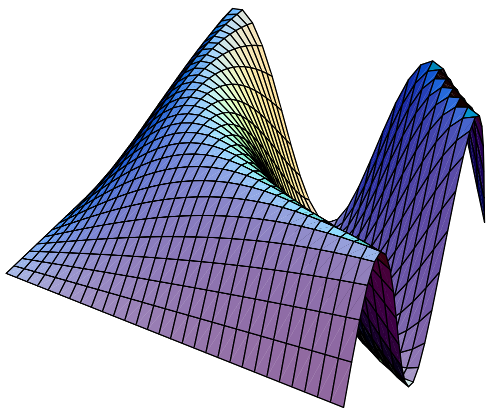
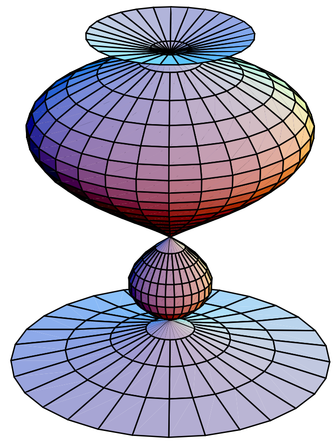


Enter I,II,III,IV,V,VI here	Equation
	$x^2 - y^2 - z^2 = 1$
	$x^2 + 2y^2 = z^2$
	$2x^2 + y^2 + 2z^2 = 1$
	$x^2 - y^2 = 5$
	$x^2 - y^2 - z = 1$
	$x^2 + y^2 - z = 1$

Problem 2c) (4 points)

Match the parametric surfaces with their parameterization. No justification is needed.





I

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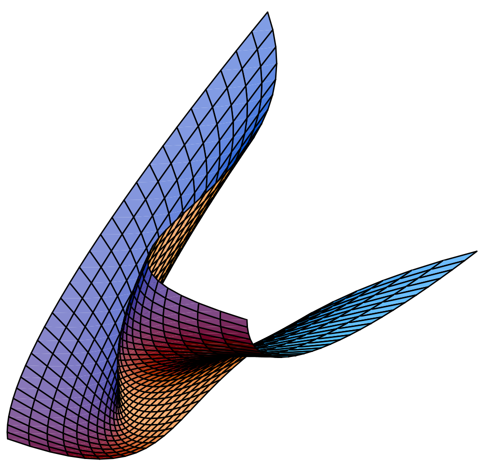

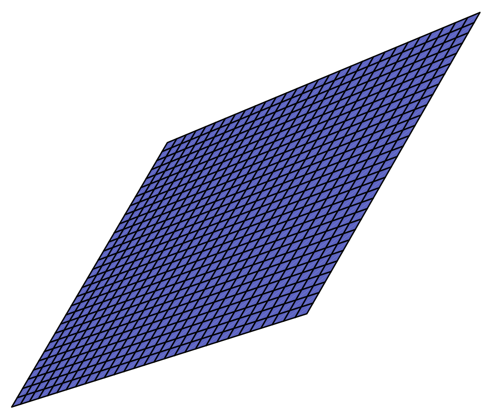

III

Enter I,II,III,IV here	Parameterization
	$\vec{r}(u,v) = \langle u, v, u+v \rangle$
	$\vec{r}(u,v) = \langle u, v, \sin(uv)) \rangle$
	$\vec{r}(u,v) = \langle 0.2 + u(1-u^2)\cos(v), (0.2 + u(1-u^2))\sin(v), u \rangle$
	$\vec{r}(u,v) = \langle u^3, (u-v)^2, v \rangle$

Problem 3) (10 points)

a) (6 points) Find a parameterization of the line of intersection of the planes $3x - 2y + z = 7$ and $x + 2y + 3z = -3$.

b) (4 points) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing that line.

Problem 4) (10 points)

a) (4 points) Find the area of the parallelogram with vertices $P = (1, 0, 0)$ $Q = (0, 2, 0)$, $R = (0, 0, 3)$ and $S = (-1, 2, 3)$.

b) (3 points) Verify that the triple scalar product has the property $[\vec{u}+\vec{v}, \vec{v}+\vec{w}, \vec{w}+\vec{u}] = 2[\vec{u}, \vec{v}, \vec{w}]$.

c) (3 points) Verify that the triple scalar product $[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$ has the property

$$|[\vec{u}, \vec{v}, \vec{w}]| \leq ||\vec{u}|| \cdot ||\vec{v}|| \cdot ||\vec{w}||$$

Problem 5) (10 points)

Find the distance between the two lines

$$\vec{r}_1(t) = \langle t, 2t, -t \rangle$$

and

$$\vec{r}_2(t) = \langle 1 + t, t, t \rangle \ .$$

Problem 6) (10 points)

Find an equation for the plane that passes through the origin and whose normal vector is parallel to the line of intersection of the planes $2x + y + z = 4$ and $x + 3y + z = 2$.

Problem 7) (10 points)

The intersection of the two surfaces $x^2 + \frac{y^2}{2} = 1$ and $z^2 + \frac{y^2}{2} = 1$ consists of two curves.

a) (4 points) Parameterize each curve in the form $\vec{r}(t) = (x(t), y(t), z(t))$.
b) (3 points) Set up the integral for the arc length of one of the curves.
c) (3 points) What is the arc length of this curve?

Problem 8) (10 points)

a) (6 points) Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = \langle -\cos(t), \sin(t), -2t \rangle$ at the point $\vec{r}(0)$.

b) (4 points) Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = \langle -\cos(5t), \sin(5t), -10t \rangle$ at the point $\vec{r}(0)$.

Hint. Use one of the two formulas for the curvature

$$\kappa(t) = \frac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|} = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3} \ ,$$

where $\vec{T}(t) = \vec{r}\ '(t)/|\vec{r}\ '(t)|$. The curvatures in b) can be derived from the curvature in a). There is no need to redo the calculation in b) if you give a proper justification.

Problem 9) (10 points)

For each of the following, fill in the blank with $<$ (less than), $>$ (greater than), or $=$ (equal).

Justify your answer completely.

1. The arc length of the curve parameterized by $\vec{f}(t) = \langle \cos 2t, 0, \sin 2t \rangle$, $0 \leq t \leq \pi$. ______ The arc length of the curve parameterized by $\vec{g}(u) = \langle 3, 2\cos u^2, 2\sin u^2 \rangle$, $0 \leq u \leq \sqrt{\pi}$.

2. The arc length of the curve parameterized by $\vec{f}(t) = \langle t^2, 2\cos t, 2\sin t \rangle$, $0 \leq t \leq 2\pi$. ______ The arc length of the curve parameterized by $\vec{g}(u) = \langle u^4, 2\cos u^2, 2\sin u^2 \rangle$, $0 \leq u \leq 2\pi$.

3. The arc length of the curve parameterized by $\vec{f}(t) = \langle 1 + 3t^2, 2 - t^2, 5 + 2t^2 \rangle$, $0 \leq t \leq 1$. ______ The arc length of the curve parameterized by $\vec{g}(u) = \left\langle \frac{1}{2}u^2, u, \frac{2\sqrt{2}}{3}u^{3/2} \right\rangle$, $0 \leq u \leq 2$.

4. The arc length of the curve parameterized by $\vec{f}(t) = \langle \sin t, \cos t, t \rangle$, $1 \leq t \leq 5$. ______ The arc length of the curve parameterized by $\vec{g}(u) = \langle u \sin u, u \cos u, u \rangle$, $1 \leq u \leq 5$.

Problem 10) (10 points)

Given the plane $x + y + z = 6$ containing the point $P = (2, 2, 2)$. Given is also a second point $Q = (3, -2, 2)$.

a) (5 points) Find the equation $ax + by + cz = d$ for the plane through P and Q which is perpendicular to the plane $x + y + z = 6$.

b) (5 points) Find the symmetric equation for the intersection of these two planes.