SECOND HOURLY in 19 TWEETS **Math21a Fall 2014**

Each tweet **140** characters or less.

Cairaut: $f_{xy} = f_{yx}$ Proof:
$f_{xy} \sim h^2[(f(x+h,y+h) - f(x,y+h)) - (f(x+h,y) - f(x,y))]$
$f_{yx} \sim h^2[(f(x+h,y+h) - f(x+h,y)) - (f(x,y+h) - f(x,y))]$

ODE: equation for function f involving derivatives
PDE: equation for function f involving at partial derivatives. Example: $f_x = f_{yy}f + f^2$

Heat equation: $u_t = u_{xx}$
Wave equation: $u_{tt} = u_{xx}$
Laplace equation $u_{xx} + y_{yy} = 0$
Transport equation $u_t = u_x$
Burgers equation $u_t + uu_x = u_{xx}$

Gradients are perpendicular to level sets.
Proof: $r(t)$ on $f = c$ satisfies
$0 = d/dt f(r(t)) = \nabla f(r(t)) \cdot r'(t)$
showing that ∇f is orthogonal

Directional derivative is max in gradient direction. Proof:
$D_v f = |\nabla f \cdot v| = |\nabla f| \cos(a)$
For $w = \nabla f / |\nabla f|$,
$D_w f = |\nabla f|$

Second derivative test: discriminant
$D = f_{xx}f_{yy} - f_{xy}^2$ and $A = f_{xx}$ determine:
$D > 0, A > 0 \Rightarrow$ min, $D > 0, A < 0 \Rightarrow$ max,
$D < 0 \Rightarrow$ saddle, $D = 0$ not know.

Lagrange:
$f_x = Lg_x$
$f_y = Lg_y$
$g(x,y) = c$
Proof: $\nabla(f)$ and $\nabla(g)$ are parallel. Else moving along $g = c$ crosses level curves.

Tangent plane at P : find $\nabla f = \langle a, b, c \rangle$ and plane $ax + by + cz = d$, (get d by plugging in P). $\nabla f = \langle a, b \rangle$ gives tangent line $ax + by = d$.

Estimate $f(3.001, 4.9999)$ by computing the gradient $\langle a, b \rangle$ of f at $(3, 5)$ and get $L(3.001, 4.9999) = f(3, 5) + a \cdot 0.001 - b \cdot 0.0001$.

Type I = "bottom to top" integration on $[a, b]$ on x-axis
Type II = "left to right" integration on $[c, d]$ on y-axis

Double integral $\int\int_R f(x, y)dxdy$ interpretation: signed volume under the graph of f. This is volume if $f \geq 0$.

Fubini: for rectangular regions only:
$\int_a^b \int_c^d f(x, y)dxdy = \int_c^d \int_a^b f(x, y)dydx$

Surface area of a parametrized surface $r(u, v)$, defined for a region R is $\int\int_R |r_u \times r_v| dudv$.

Polar integration:
include an integration factor r.
Proof: $\vec{r}(s, t) = \langle r\cos(t), r\sin(t), 0 \rangle$, $|\vec{r}_r \times \vec{r}_t| = r$.

By parts: $\int u dv = uv - \int v du$
Proof: integrate $uv' + vu' = (uv)'$
Example: $\int x\cos(x)dx = x\sin(x) - \int 1 \cdot \sin(x)dx = x\sin(x) + \cos(x) + C$

Substitution: Example:
$\int x^4 \exp(x^5)dx$
$u = x^5, du = 5x^4dx$ $\int \exp(u)/5du = \exp(x^5)/5 + C$

Tips for 2D integrals: make a picture, consider other coordinates and change order of integration.

Helpful identities:
$\cos^2(t) + \sin^2(t) = 1$
$\cos^2(t) = (1 + \cos(2t))/2$
$\sin^2(t) = (1 - \cos(2t))/2$
$\sin(t)\cos(t) = \sin(2t)/2$

$\int x^n dx = x^{n+1}/(n + 1)$
$\int \exp(ax)dx = \exp(ax)/a$
$\int \cos(ax) = \sin(ax)/a$
$\int \sin(ax) = -\cos(ax)/a$
$\int dx/x = \log(x)$
$\int dx/(1 + x^2) = \arctan(x)$

Oliver Knill, November 2, 2014.