Homework 24: Triple integrals

This homework is due Wednesday, 11/5 rsp Tuesday 11/11.

1 Evaluate the iterated integral

$$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} 12 dx\, dz\, dy \ .$$

2 Evaluate the triple integral

$$\int\int\int_E yz\cos\left(x^5\right) dV \ ,$$

where

$$E = \{(x,y,z) \mid 0 \le x \le \pi/2,\ 0 \le y \le x,\ x \le z \le 2x\} \ .$$

3 Evaluate the triple integral

$$\int\int\int_E xy\, dV \ ,$$

where E is bounded by the parabolic cylinders $y = 3x^2$ and $x = 3y^2$ and the planes $z = 0$ and $z = x + y$.

4 Use a triple integral to find the volume of the given solid enclosed by the paraboloid $x = y^2 + z^2$ and the plane $x = 25$.

5 Find the moment of inertia

$$\int\int\int_E (x^2 + y^2)\ dxdydz$$

about the z-axis of the solid cone $E : \sqrt{x^2 + y^2} \le z \le 10$.

Main definitions

If $f(x, y, z)$ is a function of three variables and E is a **solid region** in space, then $\int\int\int_E f(x, y, z)\ dV$ is defined as the $n \to \infty$ limit of the Riemann sum

$$\frac{1}{n^3} \sum_{(\frac{i}{n}, \frac{j}{n}, \frac{k}{n}) \in E} f(\frac{i}{n}, \frac{j}{n}, \frac{k}{n}) \ .$$

As in two dimensions, $dA = dxdy$ is the symbol for a small area, $dV = dxdydz$ reads as a small volume.

If $f(x, y, z) = 1$ then $\int\int\int_E 1\ dxdydz$ is the volume of the solid

In multivariable calculus we often encounter situations, where the triple integral is reduced to a double integral

$$\int\int_R [\int_{g(x,y)}^{h(x,y)} f(x, y, z)\ dz]\ dxdy \ .$$

For example, if $g(x, y) = 0$ and $f(x, y, z) = 1$, then

$$\int\int_R [\int_0^{h(x,y)} 1\ dz]\ dxdy = \int\int_R h(x, y)\ dxdy$$

is the signed volume of the solid under the graph of h.