

Homework 15: Directional Derivatives

This homework is due Monday, 10/19 rsp Tuesday 10/20.

- 1 Find the gradient of

$$f(x, y, z) = \sqrt{x + yz} .$$

at the point $P = (1, 3, 1)$ and use it to find the rate of change of f at P in the direction of the vector $\vec{u} = \langle 2/7, 3/7, 6/7 \rangle$.

- 2 a) (5 points) Find the directional derivative of the function $f(x, y) = \log(x^2 + y^2)$ at the point $P = (2, 1)$ in the direction of the vector $\vec{v} = \langle -1, 2 \rangle$. (We use the notation $\log = \ln$).

b) (5 points) Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at the point $P = (1, -1, 3)$ in the direction from P to $Q = (2, 4, 5)$.

- 3 a) Find the direction of steepest descent for $f(x, y, z) = \frac{(x+y)}{z}$ at the point $P = (1, 1, -1)$.

b) Find the value of the maximal rate of change at $(1, 1, -1)$ in that direction found in a). This is the directional derivative in that direction.

- 4 Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at the point $(0, 2)$ has the value 1.

- 5 [Arlington-Belmont-Waltham-Cambridge]

On <http://goo.gl/fhY1rl>,

you find a map of some suburbs of Boston (an original copy in office 432).

a) The map contains some creeks. Find an example which confirms the rule that water crosses level curves perpendicularly.

- b) The map shows also some railway tracks. Identify a railway which follows the level curves of the height.
- c) Find a point on the map where the slope is maximal in some direction. Estimate the steepness by counting level curves.

Main definition:

If f is a function of several variables and \vec{v} is a unit vector then $D_{\vec{v}}f = \nabla f \cdot \vec{v}$ is the **directional derivative** of f in the direction \vec{v} .

For $\vec{v} = \nabla f / |\nabla f|$, the directional derivative is

$$D_{\vec{v}}f = \nabla f \cdot \nabla f / |\nabla f| = |\nabla f| ,$$

so that f **increases** in the direction of the gradient. The value $|\nabla f|$ is the maximal slope.

