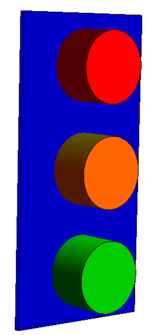
Homework 17: Lagrange multipliers

This homework is due Friday, 10/23 rsp Tuesday 10/27.

1 Use Lagrange multipliers to find the maximal value of the function $f(x, y) = 3e^{xy}$ subject to the constraint $x^3 + y^3 = 16$.

2 The material to build a traffic light is $g(x, y) = 6 + 6\pi xy + 3\pi x^2 = 12$ is fixed (the radius of each cylinder is x and the height is y and the constant 6 is the material for the back plate). We want to build a light for which the shaded region with volume $f(x, y) = 3\pi x^2 y$ is maximal. Use the Lagrange method.



3 Use Lagrange multipliers to find the maximum and minimum f under the two constraints:

$$f(x, y, z) = 3x - y - 3z;$$

$$g(x, y, z) = x + y - z = 0$$

$$h(x, y, z) = x^2 + 2z^2 = 1 \, .$$

4 Use Lagrange multipliers to prove that the triangle with maximum area that has a given perimeter p is equilateral. *Hint:* Use Heron's formula for the area:

$$A = \sqrt{s(s - x)(s - y)(s - z)}$$

where $s = p/2$ and x, y, z are the lengths of the sides.

5 Which pyramid of height h over a square $[-a,a]\times[-a,a]$ with surface area is $4a\sqrt{h^2+a^2}+4a^2=4$ has maximal volume $V(h,a)=4ha^2/3$? By using new variables (x,y) and multiplying V with a constant, we get to the equivalent problem to maximize $f(x,y)=yx^2$ over the constraint $g(x,y)=x\sqrt{y^2+x^2}+x^2=1$. Use the later variables.

Main definitions

The system of equations $\nabla f(x,y)=\lambda\nabla g(x,y), g(x,y)=0$ for the three unknowns x,y,λ are called **Lagrange equations**. The variable λ is a **Lagrange multiplier**.

$\nabla f(x,y,z)=\lambda\nabla g(x,y,z), g(x,y,z)=0$ are the **Lagrange equations** in three dimensions.

$\nabla f(x,y,z)=\lambda\nabla g(x,y,z)+\mu\nabla h(x,y,z), g(x,y,z)=0, h(x,y,z)=0$ are the **Lagrange equations** in three dimensions with two constraints. There are two Lagrange multipliers λ,μ.

Lagrange theorem: Maxima or minima of f on the constraint $g=c$ are either solutions of the Lagrange equations or critical points of g.